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A
PLAIN METHOD
Of determining the
PARALLAX of VENUS,
BY HER
TRANSIT over the SUN:
And from thence,
By ANALOGY,
The PARALLAX and DISTANCE of the SUN,
And of all the reſt of the PLANETS.

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By *JAMES FERGUSON.*

LONDON,
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MDCCLXI.

THE METHOD OF Finding the Diſtances of the PLANETS from the SUN.

ARTICLE I.

Concerning the mean diſtances of the planets.

1. THE proportional, or relative mean diſtances of the planets from the Sun, is already known to a ſufficient degree of exactneſs; for, the ſtated laws by which they move, and are retained in their orbits, are ſuch, that if the Earth's * mean diſtance from the Sun (whatever it be as to the number of miles) be divided into 1000 equal parts, Mercury's mean diſtance from the Sun muſt be equal to 387 of ſuch parts; Venus's mean diſtance, to 723; Mars's mean diſtance, 1524; Jupiter's, 5201; and Saturn's, 9540.—But how many miles are contained in any given number of ſuch parts, is not yet accurately known.

The † Sun's horizontal parallax is ſuch, that we are ſure his mean diſtance from the Earth is not leſs than 66 millions of Engliſh miles, nor much greater than 83; but what it really is between theſe two numbers, we know not. Moſt of our modern Aſtronomers have reckoned it 81; and if it be ſo, Mercury's mean diſtance from the

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* All the planets move about the Sun in elliptical orbits, and the Sun is in the lower focus of all their orbits. Therefore they muſt be nearer the Sun at ſome times than at others; and when they are mid-way between their greateſt and leaſt diſtances from the Sun, they are ſaid to be at their mean diſtances from him.

† The nature of parallaxes will be deſcribed in the third article.

Sun muſt be 31347000 miles; Venus's mean diſtance, 58563000; Mars's, 123444000; Jupiter's, 421281000; and Saturn's, 772740000. For, as 1000 is to 387, ſo is 81000000 to 3134700 miles, for the diſtance of Mercury from the Sun. And, as 1000 is to 723, ſo is 81000000 to 58563000 miles, for the diſtance of Venus from the Sun. Likewiſe, as 1000 is to 1524, ſo is 81000000 to 123444000 miles for the diſtance of Mars from the Sun; and ſo on to Jupiter and Saturn.

3. As it is a problem of the higheſt importance in Aſtronomy, to find the Sun's diſtance from the Earth, ſince by this we ſhall be able to aſcertain the dimenſions of our whole SOLAR SYSTEM; it may be preſumed, that a plain account of the method propoſed by Dr. HALLEY for that purpoſe, by the tranſit of Venus over the Sun on the 6th of June 1761, will not be unacceptable to the public.

4. The Doctor's Diſſertation on that ſubject is printed in Latin, in *Motte*'s Abridgment of the Philoſophical Tranſactions (Vol. I. pag. 243.) and ſhall be tranſlated into Engliſh in this Treatiſe, for the ſake of thoſe who do not underſtand the Latin. But as the ſaid Diſſertation is ſhort, and was written only for thoſe who are well ſkilled in Aſtronomy, to whom it is abundantly plain; yet, any one who would endeavour to make it plain to others, muſt premiſe ſeveral things which the Doctor has omitted.

5. In conſequence of this, I ſhall begin with ſhewing how to find the altitudes, and angular diſtances, of the heavenly bodies; and then explain the nature of parallaxes in general, with the method of finding them by obſervation: which will be eaſily underſtood by thoſe who have learnt the elements of plain trigonometry.

ARTICLE II.

Shewing how to find the altitudes and angular diſtances of the celeſtial bodies.

6. Let a large quadrant, as *ABC* (Plate I. Fig. 1.) be ſo adjuſted to a ſtrong upright pillar *EF*, by the two rings *g* and *h*, that the quadrant may be turned round the pillar without ſhaking: and in being ſo turned round, its edge *BC* may always be perpendicular to the plane of the horizon *CI*. Let a teleſcope as *CD*, with croſs hairs in it, be made moveable on *C*, the center of the quadrant; and let an

index, as *e*, be fixed to the end *D* of the teleſcope, to point out the diviſions on the graduated limb *AfB*.

7. Let *ISZ* be a quarter of a vertical circle in the ſtarry heaven, the plane of which circle is perpendicular to the plane of the horizon *CI*; for the altitude of every celeſtial object above the horizon, is taken in a circle, whoſe plane is perpendicular to the horizon.—Now, as every circle, be it great or ſmall, contains 360 degrees, a quarter of a circle muſt contain 90. And ſince all the planets, together with the Sun and Moon, ſeem equidiſtant with the fixed ſtars from us, we refer their places to the ſtarry heaven, the diſtance of which is ſo immenſely great, that the whole Earth is but a dimenſionleſs point in proportion to that diſtance. And therefore, we may conſider the center of the quadrant to be in the center of the ſtarry firmament, let it be placed upon any part of the Earth whatever.

8. It is evident, that when any celeſtial body is in the plane of the horizon, as at *I*, it can only be ſeen through the teleſcope by an eye placed at *D*, when the teleſcope is in the horizontal poſition *AC* on the quadrant; and then, the index *e* will point to 0 on the graduated limb *AfB*, ſhewing that the celeſtial object has then no altitude above the horizon. If the object be in the zenith at *Z*, it is 90 degrees above the horizon, and the teleſcope muſt be in the upright poſition *BC*, when the object is ſeen in the croſs hairs by an eye placed at *D*; and then, the index will be at the 90th degree of the quadrant. Conſequently, as all the degrees of the celeſtial arc *ISZ* correſpond to the degrees of the quadrant, whenever any celeſtial object is ſeen through the teleſcope, the index *e* muſt be on the ſame degree, or part of a degree, on the quadrant, that anſwers to the altitude of the object above the horizon. Thus, if the Moon *M* be ſeen through the teleſcope, when ſhe is 20 degrees above the horizon, referred to *m* in the arc *ISZ*, the index *e* muſt be on the 20th degree of the quadrant: and if a ſtar, as *S*, be ſeen through the teleſcope, when it is 45 degrees above the horizon, the teleſcope muſt be in the poſition *fC* to ſee it, and the index *e* will be on the 45th degree of the quadrant.

9. When any two given bodies are in the ſame vertical circle; that is, when a plumb-line would hide them both from the eye at once, if their altitudes be taken, and the leſſer altitude be ſubtracted from the greater, the remainder will be their angular diſtance from each other. Thus, in the above caſe, if the Moon be 20 degrees high, and the ſtar 45, the difference *mS* will be 25 degrees; which is the Moon's angular diſtance from the Star.

10. When

10. When the two given bodies are not in the ſame vertical circle; that is, when a plumb-line will not hide both their centers at once from the obſerver's eye, but that a circle paſſing through them both is oblique to the horizon; we muſt uſe a quadrant (or perhaps a ſemicircle) which turns on a ball and ſocket, like a common theodolite for ſurveying land; and fix it ſo, that its face may be in the ſame plane with both the bodies: then, looking at each body ſeparately through the teleſcope, mark what degree, or part of a degree, the index is at on the graduated limb, when each body is ſeen; and the arc of the inſtrument, which lies between thoſe points where the index ſtood at each obſervation, is the angular diſtance of the bodies from each other.

11. The larger the inſtrument is, the better; for then, each degree may be divided into 60 equal parts or minutes, either by diagonal lines, or by (what is called) a *nonius* diviſion. On ſome large quadrants, the obſerver can now reckon to four or five ſeconds of a minute, by a particular contrivance lately added to the index of the teleſcope.

ARTICLE III.

Concerning parallaxes, and the method of finding the diſtances of the Sun and Moon thereby.

12. If a candle be placed on a table in the middle of a room, and viewed by two perſons ſitting at oppoſite ſides of the chimney; whatever place of the wall is hid by the flame of the candle from either perſon, a different place will be hid by it from the other: ſo that, if they refer the flame to the wall, its place thereon will be different to each obſerver. The farther they are from each other, the greater will be the diſtance of the places on the wall to which they refer the place of the flame, and *vice verſâ*. This apparent change of place on the wall, may be called the parallax of the flame; for the parallax of the Sun or Moon in the heavens (which are objects between us and the ſtars) is no other than its apparent change of place among the ſtars, as ſeen at the ſame inſtant from different places of the Earth's ſurface. And ſince the apparent places of the Sun or Moon are different in the heavens, with regard to the fixed ſtars, when viewed by two or more obſervers at the * ſame inſtant of time, on different

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* The reaſon for mentioning *the ſame inſtant of time* is, becauſe the Sun and Moon are in motion, and the ſtars are at reſt. For if the place of the Sun or Moon were not taken at the ſame inſtant, by both the obſervers, it would be different, even though the Sun and Moon were at the ſame diſtance as the ſtars are.

parts of the Earth's ſurface, it evinces them to be nearer to us than the ſtars are. For, if their diſtance were the ſame as that of the ſtars, their apparent places could not be different among the ſtars; as may be eaſily conceived by ſetting the candle cloſe to the wall, and then its flame will hide the ſame ſpot thereon from both the ſpectators ſitting by the chimney.

13. If the candle be removed from the wall toward theſe ſpectators, then, if one raiſes his head above the level of the other's, he will ſee the flame lower on the wall than the other perſon ſees it; if he ſtoops his head, the flame will appear higher.

14. But if two perſons ſtand at oppoſite ſides, in one end of a long avenue, and look at a third perſon ſtanding at the other end, between them and a wall at a diſtance beyond him, he will ſo nearly hide the ſame place of the wall from both the others, that they will ſcarce perceive any difference; becauſe their diſtance from each other ſcarce bears any ſenſible proportion to his diſtance from them both.—Hence we infer, that the farther diſtant the Sun, or Moon, or any planet, is from the Earth, the leſs will its parallax be in the heavens, as ſeen from different places of the Earth. And we ſhall ſoon prove, that the diminution of the parallax is in direct proportion to the diſtance of the object.

15. When the Moon is at her mean diſtance from the Earth, her horizontal parallax is found to be * 58′ 10″ (or 3490″), and the Earth's ſemidiameter is known to be 4000 Engliſh miles; which two things being given, the Moon's mean diſtance from the Earth's center may be found by the following analogy, from *Gardiner*'s logarithmic tables.

As the tangent of the horizontal parallax 58′ 10″ —	8.228442
Is to the log. of unity, or the Earth's ſemidiameter —	0.000000
So is radius, or the ſine of 90° — — —	10.000000
To the log. of the Moon's diſtance 59.96 —	1.771558

Then 59.96 ſemidiameters of the Earth being multiplied by 4000, the number of miles contained in the Earth's ſemidiameter, gives 239840 miles for the Moon's diſtance from the Earth.

16. By the ſame analogy, the Sun's diſtance from the Earth is found, when his horizontal parallax is given. And here we ſhall compute his diſtance, upon ſuppoſition that his horizontal parallax is either 10″, 10½″, 11″, 11½″, 12″, or 12½″.

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* *N. B.* This mark ′ ſignifies minutes of a degree, and this mark ″ ſignifies ſeconds of a minute.

But becauſe the logarithm of unity (or of 1) is nothing, we may leave it out; and then, by ſubtracting the logarithmic tangent of the parallax from the radius, (which is 10) the remainder will be the logarithm of the Sun's diſtance in ſemidiameters of the Earth. Therefore,

I. If the Sun's parallax be 10″,	From the radius —	10.000000
	Subtr. the tang. of 10″	5.685575
	Remains — —	4.314425, which, in

the tables, is the log. of 20626.5. Then, 20626.5 ſemidiameters of the Earth being multiplied by 4000, the number of miles in the Earth's ſemidiameter, gives 82506000 miles for the Sun's diſtance from the Earth.

II. If the Sun's parallax be 10½″,	From the radius	10.000000
	Subtr. the tang. of 10½″	5.706271
	Remains — —	4.293729, namely,

the log. of 19666.6; which being multiplied by 4000, gives 78666400 miles for the Sun's diſtance from the Earth.

III. If the Sun's parallax be 11″,	From the radius —	10.000000
	Subtr. the tang. of 11″	5.726968
	Remains — —	4.273032, the log.

of 18751.3; which being multiplied by 4000, gives 75005200 miles for the Sun's diſtance from the Earth.

IV. If the Sun's parallax be 11½″,	From the radius —	10.000000
	Subtr. the tang. of 11½″	5.745862
	Remains — —	4.254138, the log. of

17953.0; which being multiplied by 4000, gives 71812000 miles for the Sun's diſtance from the Earth.

V. If the Sun's parallax be 12″,	From the radius —	10.000000
	Subtr. the tang. of 12″	5.764756
	Remains — —	4.235244, the log.

of 17188.7; which being multiplied by 4000, gives 68754800 miles for the Sun's diſtance from the Earth.

VI. If the Sun's parallax be 12½″,	From the radius —	10.000000
	Subtr. the tang. of 12½″	5.782137
	Remains — —	4.217863, the log.

of 16514.4; which being multiplied by 4000, gives 66057600 miles for the Sun's diſtance from the Earth.

17. Now,

Pl. I.

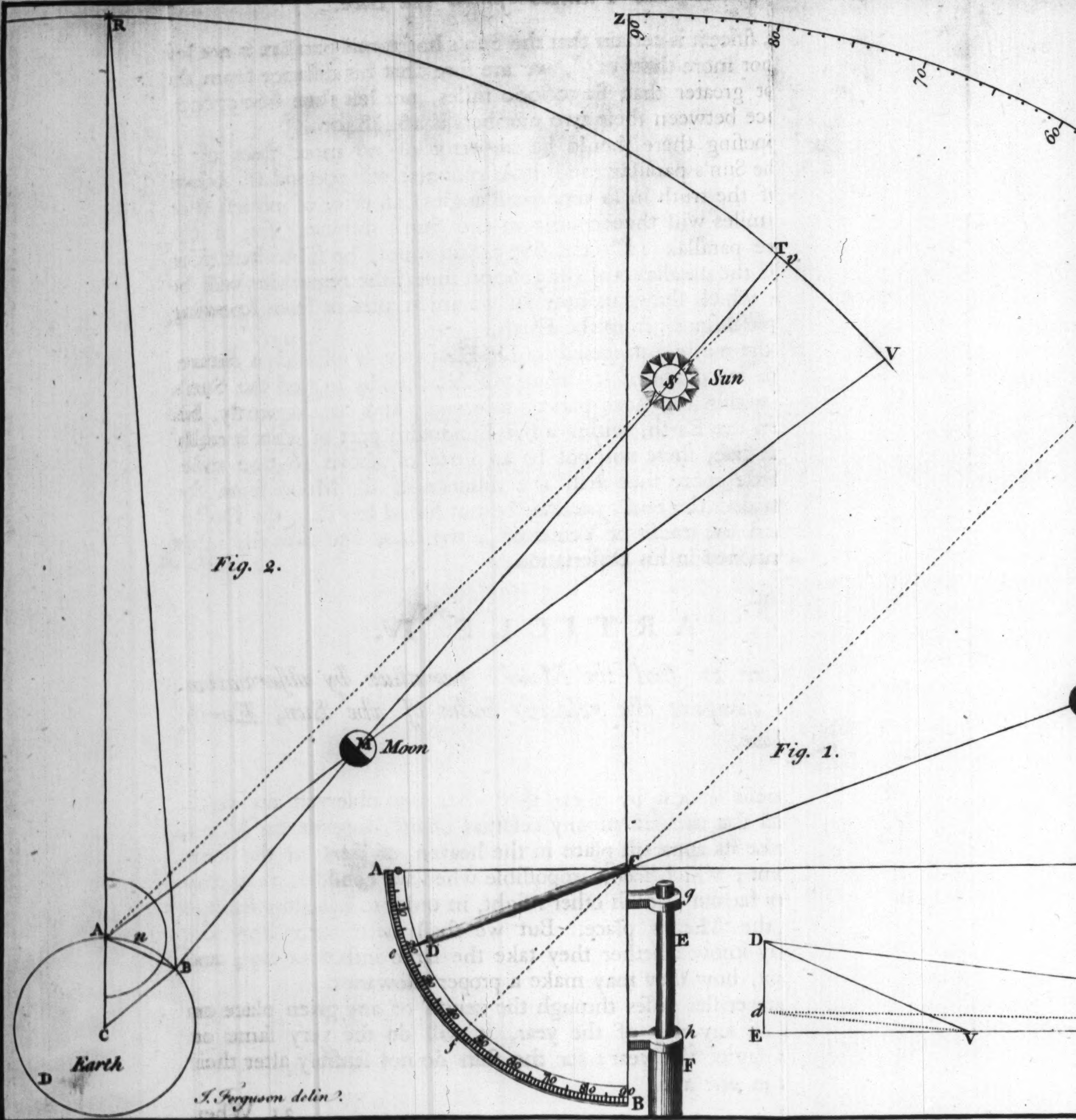

Fig. 2.

Fig. 1.

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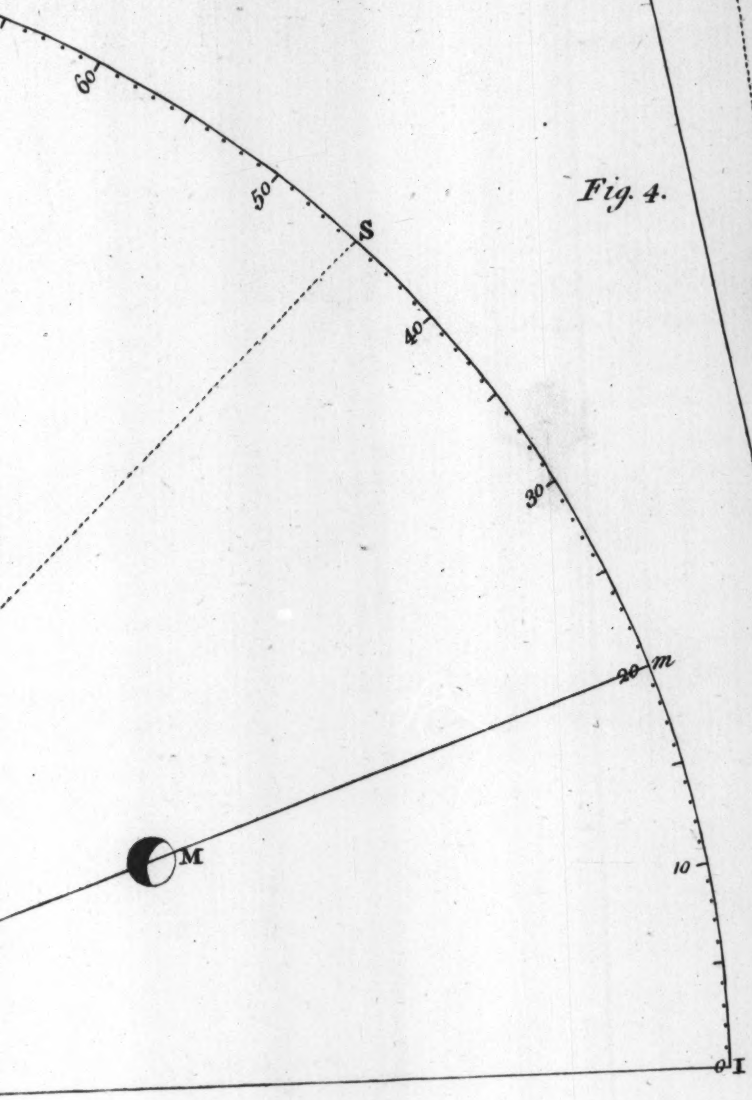


Fig. 4.

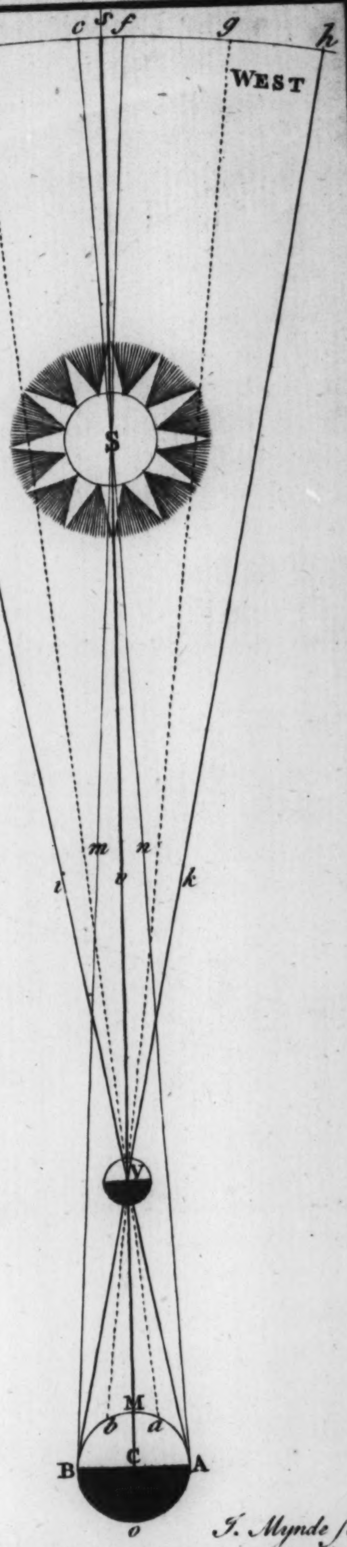
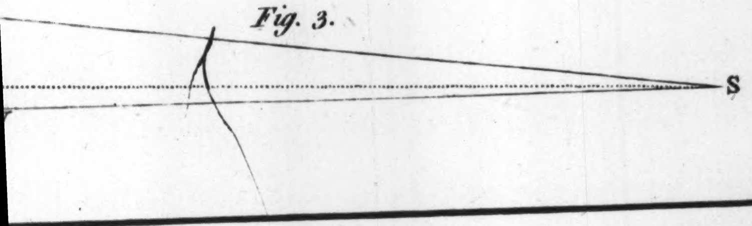


Fig. 3.



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17. Now, ſince it is certain that the Sun's horizontal parallax is not leſs than 10″, nor more than 12½″, we are ſure that his diſtance from the Earth is not greater than 82506000 miles, nor leſs than 66057600; the difference between theſe two numbers is 16448400.

And ſuppoſing there ſhould be an error of no more than 2″ in obſerving the Sun's parallax (and no Aſtronomer will pretend to obſerve within 2″ of the truth in ſo very ſmall angles) an error of no leſs than 13751200 miles will thence ariſe in the Sun's diſtance: for, if the reſult of the parallax 12″ (viz. 68754800 miles) be ſubtracted from the reſult of the parallax 10″ (82506000 miles) the remainder will be 13751200; which ſhews us how far we are at preſent from knowing the Sun's true diſtance from the Earth.

18. But the method propoſed by Dr. Halley is of ſuch a nature, that he made no doubt of its being poſſible thereby to find the Sun's parallax to within the 40th part of a ſecond; and, conſequently, his diſtance from the Earth, within a five hundredth part of what it really is; in which caſe, there will not be an error of above 164000 miles (which is little more than half the diſtance of the Moon from the Earth), provided the Sun's parallax be not found leſs than the Doctor ſuppoſed, and the tranſit of Venus be in the ſame line over the Sun's diſc, as mentioned in his Diſſertation.

ARTICLE IV.

Shewing how to find the Moon's parallax by obſervation, and to compute the relative bulks of the Sun, Earth, and Moon.

19. It would appear by § 12, that when two obſervers are determined to find the parallax of any celeſtial object, ſuppoſe the Moon, they muſt take its apparent place in the heaven of ſtars, at the ſame inſtant of time; which ſeems impoſſible when we conſider, that they muſt be very far out of each other's ſight, in order to have any ſenſible parallax in the Moon's place. But we ſhall here ſhew how the obſervers may know whether they take the ſame inſtant or no; and if they do not, how they may make a proper allowance.

20. Whatever ſtar paſſes through the zenith of any given place on the Earth, on any day of the year, it will do the very ſame on every other day of the year: for the ſtars do not ſenſibly alter their declinations in one age.

21. When the Moon's place in the ecliptic is found, which may be eaſily done to a ſufficient degree of exactneſs for our preſent purpoſe, the quantity of her declination in any part of the day is ſoon found. But theſe things are computed to our hand in almoſt every Ephemeris.

22. Two very fit places for obſervation would be *Vienna* and the *Cape of Good Hope*, being both on the ſame meridian, and 82° 50′ diſtant from each other: the latitude of the former is 48° 20′ north, and that of the latter 34° 30′ ſouth.—Now, if both the obſervers are on one and the ſame meridian, and make their obſervations when the Moon is on their meridian, they will do this at the ſame inſtant of time.

23. If the meridians of the places do differ, and the obſervers know the quantity of that difference, they can eaſily allow for the Moon's declination, whilſt ſhe is paſſing from the meridian of one place to that of the other. And they may find the difference of their meridians exactly by the eclipſes of Jupiter's ſatellites.

But they need not be very ſcrupulous about the difference of their meridians, if they agree to obſerve upon a day when the Moon is in either of the tropics, which they have twice in every month of the year. For then, a few degrees difference of longitude of their places will not ſenſibly alter the Moon's declination between their meridians.

24. This point being ſettled, let us ſuppoſe *ABD* (Plate I. Fig. 2.) to be the Earth; *A* the place of one obſerver, and *B* the place of the other; which ought not to be leſs than 40 or 50 degrees from each other, and if more remote, ſo much the better. Let *M* be the Moon, and *RTV* an arc of the ſtarry heavens.—Let each obſerver find the latitude of his place, and when they meet again, they will know by their latitudes how far their places of obſervation are from each other, provided both were on the ſame meridian; becauſe the length of a degree, or any number of degrees on the Earth's ſurface, is known. If their meridians are ſomewhat different, yet the known difference of their latitudes will anſwer the purpoſe; becauſe any two given parallels of latitude are equidiſtant from each other, quite round the Earth: and therefore, all the chords of an arc of the Earth's ſurface, from parallel to parallel, are equal to one another, if they are taken at right angles to theſe parallels.

25. Let each obſerver take the angular diſtance of the Moon on his meridian, from that point of it through which he obſerved a given ſtar *R* to paſs; which we ſhall call the Moon's angular diſtance from *R*. Then it is evident by the figure, that when the Moon is on the meridian of the obſerver at *A*, ſhe will appear to him to be at *V* in

the ſtarry heaven: and therefore the angle *RAV*, whoſe meaſure is the arc *RV*, is the Moon's angular diſtance from *R*, as ſeen by him in the right line *AMV*. But the obſerver at *B* will then ſee the Moon as at *T* in the heavens, in the right line *BMT*; and the angle *RBT*, whoſe meaſure is the arc *RT*, is the Moon's angular diſtance from *R*, as ſeen by the obſerver at *B*.

26. When the two obſervers meet again, *A* tells *B* that the angular diſtance to him was the arc *RV*; and *B* tells *A* that to him the angular diſtance was the arc *RT*. Now, as both theſe arcs are given by obſervation, ſubtract *RT* from *RV*, and the remainder will be the arc *TV*, which is the Moon's parallax, or apparent difference of place in the ſtarry heavens, as ſeen from the places *A* and *B*; which ought to be on different ſides of the equator, and as nearly at equal diſtances from it as may be, on account of the Earth's ſpheroidical figure. But a few degrees of difference will not be of any bad conſequence.

27. Now, as the Moon, by reaſon of her parallax, will be ſeen by the above obſervers at different diſtances from the zenith of either of their places; that is, in different parallels of declination, her horizontal parallax may be found by the following analogy.

As the ſum of the ſines of the two diſtances of the Moon from the zenith, is to radius; ſo is the diſtance of the two apparent parallels of declination, to the Moon's horizontal parallax: which being thus found, her diſtance from the Earth's center may be found by § 15.

28. The Moon's diſtance from the two places *A* and *B* may be found in the following manner.

Draw the two right lines *VMA* and *TMB*, croſſing each other in the Moon's center at *M*; and they will form the two equal and oppoſite angles *TMV* and *AMB*, either of which is the Moon's parallax, whoſe meaſure is the arc *TV*.

Draw *AC* and *BC*, each whereof is a ſemidiameter of the Earth; and join *AB*, which is the chord of the given arc *AnB*, or leaſt diſtance between the parallels of latitude of the places *A* and *B*.

In the triangle *AMB*, the angle at *M* is given; and ſo is the ſide *AB*, becauſe it is the chord of the given arc *AnB*. The ſides *AM* and *BM* are to be found.

Becauſe *RAC* is a right line, and the angular points of the three angles *RAM*, *MAB*, and *BAC* all meet in the point *A*, it is plain that the ſum of theſe three angles is equal to two right ones, that is, to 180°: for they include a ſemicircle, the center of which is at *A*. But the angle *RAV* is given, being the Moon's angular diſtance from *R*, as ſeen by the obſerver at *A*.

Likewiſe,

Likewiſe, in the iſoſceles triangle *ACB*, the angle *CAB* is given, being half the ſupplement of the given angle *ACB* to 180°. Take therefore the ſum of the given angles *RAV* and *CAB* from 180°, and there will remain the angle *MAB*, which added to the given angle *AMB*, and the ſum taken from 180°, remains the angle *ABM*.

29. Now ſay, As the ſine of the angle *AMB*, is to the ſine of the angle *MAB* (and of *MBA*), ſo is *AB*, the chord of the given arc *AnB*, to the diſtance *BM* (and *AM*), which are the Moon's diſtances from *A* and *B*.

30. This method ſucceeds very well in finding the Moon's diſtance from the Earth, becauſe her parallax is very ſenſible: but it is altogether uncertain in finding the Sun's diſtance, becauſe the arc *Tv*, which meaſures the Sun's parallactic angle *ASB* (equal to *TSv*), is too ſmall to be accurately determined by means of the beſt inſtruments that ever were made for obſervations.

31. According to the beſt modern obſervations, the Sun's horizontal parallax is about $10\frac{1}{2}''$; and if this be its exact quantity, we have the Sun's diſtance from the Earth, as in § 16, 78666400 Engliſh miles.

32. Now, ſince the horizontal parallax of any celeſtial object is equal to the angle which the Earth's ſemidiameter ſubtends as ſeen from that object, if the Sun's horizontal parallax be $10\frac{1}{2}''$, the whole diameter of the Earth, as ſeen from the Sun, will ſubtend an angle of 21″, that is, double the Sun's horizontal parallax.—But the Sun's mean diameter, as ſeen from the Earth, is 32′ 11″ (or 1931″): therefore the Sun's diameter is to the Earth's diameter, as 1931 is to 21. But the Earth's diameter is 8000 Engliſh Miles. Therefore, as 21 is to 1931, ſo is 8000 to 736000. So that if the Sun's horizontal parallax is $10\frac{1}{2}''$, his diameter muſt be 736000 Engliſh miles. And ſince the diſtances of all the planets from the Earth, in any part of their orbits, may be found when the Sun's diſtance is known, the angle under which the Earth's diameter would appear, as ſeen from the planets, may be eaſily inveſtigated: and then, by meaſuring their diameters, as ſeen from the Earth, with a good micrometer, their real diameters may be found by comparing them with the diameter of the Earth.

33. And ſince the relative bulks of ſpherical bodies are in ſuch proportion as the cubes of their diameters, and the Sun's diameter is 92 times as great as the Earth's, his bulk muſt be 778688 times as great as the Earth's.

34. The Moon's horizontal parallax, at her mean diſtance from the Earth, is 58′ 10″ (or 3490″): therefore, the Earth's diameter, as ſeen from the Moon, being double the Moon's horizontal parallax, muſt

ſubtend an angle of 1° 56′ 20″ (or 6980″). But the Moon's diameter, as then ſeen from the Earth, ſubtends an angle of 31′ 34″ (or 1894″); ſo that the Earth's diameter is to the Moon's diameter, as 6980 to 1894. Therefore, as 6980″ is to 8000 miles (the Earth's real diameter) ſo is 1894 to 2171 miles very nearly, which is the Moon's real diameter. Hence, the Earth's diameter is to the Moon's, as 3.68 is to 1; and conſequently, the Earth's bulk is to the Moon's bulk, as 49.8 to 1.

35. And thus, ſince the apparent diameters of the Sun, Moon, and planets, may be meaſured by a good micrometer; if their horizontal parallaxes are given, their real diſtances, diameters, and bulks may be found.——But, from the harmony of the whole ſyſtem (§ 1.) it appears, that when we can find the Sun's parallax, we may thence find the parallaxes of all the planets, together with their real diſtances, diameters, and magnitudes.

ARTICLE V.

Concerning the horizontal parallaxes of the Sun and Venus, and the parallax of Venus from the Sun.

36. Since the relative diſtances of all the planets from the Sun are known (§ 1.), their relative diſtances from the Earth are alſo known when they are in conjunction with the Sun. And therefore (as was ſaid above) if the parallax of any planet can be found, the parallaxes of all the reſt are found of courſe; becauſe the parallaxes are in the inverſe proportion of the diſtances *.

Thus, the Earth's mean diſtance from the Sun being 1000 (by § 1.) and Venus's mean diſtance from the Sun being 723, if 723 be ſubtracted from 1000, the remainder will be 277 for the mean diſtance of Venus from the Earth, when ſhe is between the Earth and the Sun; which ſhe will be, on the 6th of June 1761.

Therefore, if (with Doctor Halley) we ſuppoſe the Sun's horizontal parallax to be $12\frac{1}{2}''$, the horizontal parallax of Venus, on that day, will be $45\frac{12}{100}''$ very nearly; for, as 277 is to 1000, ſo is $12\frac{1}{2}$ to $45\frac{12}{100}$ nearly.

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* For by the rules of plain trigonometry, *SD* is to *VD* (Plate I. Fig. 3.) as the ſine of the angle *EVD* is to the ſine of the angle *ESD*, univerſally. But if *D* approaches towards *E* (as from *D* to *d*), theſe ſines are nearly as the angles themſelves, and *SD* (now *Sd*) is nearly equal to *SE*. Therefore the parallactic angles *EVD*, *ESD* are as *SE* to *VE*; inverſely as the diſtances *EV* and *ES*. In ſuch ſmall angles as thoſe of the parallaxes of the Sun and planets, it is all the ſame whether the ſines or tangents be uſed in working out the analogies.

37. But the Sun is a little farther from the Earth in June than in December; in March and September his diſtance is at a mean between the greateſt and the leaſt. On the 6th of June, his diſtance from the Earth is 1015, and his diſtance from Venus is 726. This difference ariſes from the elliptical figure of the orbits of the Earth and Venus.

38. Therefore, ſince Venus is then between the Earth and the Sun, ſubtract 726 from 1015, and the remainder will be 289, which is the diſtance of Venus from the Earth at that time. Then ſay, as 289 is to 1015, ſo is the Sun's horizontal parallax $12\frac{1}{2}''$ to $43\frac{9}{10}''$, which is the horizontal parallax of Venus at that time.

39. Subtract the Sun's horizontal parallax $12\frac{1}{2}''$ from the horizontal parallax of Venus $43\frac{9}{10}''$, and the remainder will be $31\frac{4}{10}''$ for the horizontal parallax of Venus from the Sun; which, in the following part of the work, we ſhall call 31″, becauſe $\frac{4}{10}$ of a ſecond is inconſiderable.

In the right line *CVS* (Fig. 4.) take *SV* to *SC* as 726 to 1015; then, *S* will be the Sun's center, *V* the center of Venus, and *C* the center of the Earth, at their relative diſtances from each other on the 6th of June 1761. Let *esb* be an arc in the ſtarry heaven, which, in fact, is ſo far diſtant from the Earth, that if the Earth were as big as its whole annual-orbit, it would appear no bigger than a point, as ſeen from the ſtars. The truth of this is evident; for, as the Earth is carried every year round the Sun, in an orbit whoſe diameter is about 160 millions of miles, the Earth is 160 millions of miles nearer ſome of the ſtars at any given time of the year, than at the oppoſite time: and yet, neither is the apparent magnitudes of theſe ſtars, nor their angular diſtances from each other, affected in the leaſt by this great change of the Earth's poſition.—But as it is impoſſible to repreſent theſe things truly in a figure; and ſince we are obliged to draw the Earth and Venus by much too big; and the Sun too ſmall, in proportion to their diſtances; we can never have the true quantities of the parallactic angles repreſented in ſuch figures. So that the uſe of theſe figures is only to give ideas of the angles in general. To have their true quantities, we muſt have recourſe to calculations.

40. This being premiſed, to avoid miſ-leading the reader's imagination with regard to the true quantities of the parallaxes of the Sun, Moon, and Venus; let *AMB* be the Earth, *V* Venus, and *S* the Sun. Whilſt the half *AMB* of the Earth is enlightened, becauſe it is turned toward the Sun, the oppoſite half *BaA* is in the dark.

41. The Earth's motion on its axis is from weſt, by ſouth, to eaſt: that is, in the direction *AMBoA*. Therefore, when the place of the obſerver revolves to *A*, the Sun *S* will be riſing to him, in the eaſtern ſide of his horizon: when the motion of his place carries him to *M*, the Sun will be on his meridian; and when it carries him to *B*, the Sun will be ſetting in the weſtern ſide of his horizon.—The like would happen with regard to Venus *V*, if ſhe had no real motion from ſun-riſe to ſun-ſet.

42. Let us ſuppoſe Venus not to move from *V* for one day, in order to conſider both her parallax and the Sun's.—To an obſerver at the Earth's center *C*, which is at reſt, the Sun's center *S* would all the day be referred to the point *s* in the ſtarry heaven: and Venus would appear to be on the Sun's center at *S* (provided ſhe had then no declination from the ecliptic); ſo that there would be no apparent change of their places in the heavens; that is, they would have no parallax either to the eaſt or to the weſt.

But to an obſerver at *A* on the Earth's ſurface, who has the Sun juſt riſing in his horizon, the Sun's center *S* would appear at *c*, a little eaſtward of its true place *s* in the celeſtial arc *esb*; being then ſeen in the right line *AnSc*: and at the ſame time, the center *V* of Venus would appear at *e* in the ſame arc, conſiderably eaſt of its true place *s* as ſeen from the Earth's center. For, in the former caſe it is ſeen in the right line *AVie*; and in the latter it is ſeen in the right line *CVSs*. Conſequently, as the Sun's center *S* would appear at *c*, and Venus's center *V* would appear at *e*, as ſeen from *A*; the arc *ce* is the diſtance of Venus's center from the Sun's as ſeen in the horizon of *A*, at the ſame time that their centers are coincident at *S* (or *s*) as ſeen from the Earth's center *C*; or from *M* on its ſurface, where both the Sun and Venus are in the meridian.

43. The angle *CSA* (which is the ſame as the oppoſite angle *cSs*) is called the Sun's horizontal parallax: and the angle *CVA* (the ſame with the oppoſite angle *eVs*) is called Venus's horizontal parallax. And *CSA* is to *CVA*, as *CV* is to *CS*; that is, the horizontal parallaxes of the Sun and Venus are to one another inverſely as their diſtances from the Earth's center. Or, in other words, the Sun's horizontal parallax *CSA* is the angle under which the Earth's ſemidiameter *CA* would appear, if ſeen from the Sun at *S*; and Venus's horizontal parallax *CVA* is the angle under which the Earth's ſemidiameter *CA* would appear, if ſeen from Venus at *V*.

44. If the Sun had no parallax at all, the angular diſtance of Venus *V* from the Sun *S*, as ſeen from *A*, would be equal to the whole hori-

zontal parallax of Venus, or to the arc *ſe*: but as the Sun has a ſmall parallax *ſc* toward the ſame ſide of the heaven that Venus's parallax *ſe* lies, when they are riſing in the eaſt; the horizontal angular diſtance of Venus from the Sun at that time is only the arc *ce*, which is the difference between the Sun's horizontal parallax and Venus's, or (as it is generally called) the horizontal parallax of Venus from the Sun.

45. The horizontal parallaxes of the Sun and Venus are of the ſame quantities when they ſet, as when they riſe: only, their parallaxes make them appear eaſtward of their true places at riſing, and weſtward thereof at ſetting. For, when the place *A* revolves to *B*, the Sun and Venus ſet to it; and although their true places *S* and *V* are at *s* when referred to the ſtarry heaven, in a right line from the Earth's center, yet the apparent place of the Sun, as ſeen from *B*, is at *f*, and the apparent place of Venus is at *h*.

46. This parallax is greateſt of all in the horizon, and leſſens gradually till the object comes to the meridian. For, when the obſerver is at *A*, a and Venus at *V* ~~in his horizon,~~ ſhe appears at *d* in the heavens, with a parallax, whoſe quantity is the angle *dVs*, the ſame as *aVC*; and when the obſerver is carried to *M* by the Earth's motion, Venus is vertical in his meridian, and appears at *s* in the heavens, without a parallax, the ſame as ſhe would do if ſeen from the Earth's center *C*: then, as he revolves toward *B*, Venus changes her parallax from *s* toward *h*, in the oppoſite ſide of the horizon.

47. Hence it is plain, that if three obſervers were placed upon the Earth, at the points *A*, *M*, *B*, and ſhould look at Venus *V* at the ſame inſtant of time, the obſerver at *A* would ſee her at *e* in the heavens, the obſerver at *M* would ſee her at *s*, and the obſerver at *B* would ſee her at *h*. And at the ſame moment, the Sun *S* would appear at *c* to the obſerver *A*, at *s* to the obſerver *M*, and at *f* to the obſerver *B*.

48. Therefore, if the Sun and Venus ſhould continue at reſt for a whole day, whilſt the obſerver was carried through the arc *AMB*, the Sun to him would ſeem to move through the arc *csf*, and Venus to move through the arc *esh*.

49. And thus it appears, that the parallax in longitude (or in right aſcenſion) of the Sun or any planet, is eaſtward from the time of its riſing until it comes to the meridian: and weſtward from the time of its being on the meridian until it ſets in the horizon: decreaſing conſtantly from its riſing till it comes to the meridian, where it is nothing; and increaſing conſtantly from the meridian to the time of its ſetting.

Having now ſaid all that I think is requiſite to be underſtood by any perſon before he reads Dr. HALLEY's paper on the tranſit of Venus, I ſhall here ſubjoin it; not doubting but that on its account, this treatiſe will be welcome to any gentleman's library.

ARTICLE VI.

Containing Dr. HALLEY'S *Diſſertation on the method of finding the Sun's parallax and diſtance from the Earth, by the tranſit of Venus over the Sun's diſc on the 6th of June* 1761. *Tranſlated from the Latin copy thereof in* Motte's *Abridgment of the Philoſophical Tranſactions, Vol. I. pag.* 243; *with additional notes.*

There are many things exceedingly paradoxical, and that ſeem quite incredible to the illiterate, which yet by means of mathematical principles may be ſolved without much difficulty. Scarce any problem will appear more hard and difficult, than that of determining the diſtance of the Sun from the Earth as near the truth as poſſible: but even this, when we are made acquainted with ſome exact obſervations, taken at times fixed upon, and choſen before-hand, will without a great deal of labour be effected. And this is what I am now deſirous to lay before this illuſtrious ſociety * (which I foretel will continue for ages) that I may explain before-hand to young Aſtronomers, who may perhaps live to obſerve theſe things, the method whereby the immenſe diſtance of the Sun may be truly obtained, to within a five hundredth of what it really is.

It is well known that the diſtance of the Sun from the Earth is by different Aſtronomers ſuppoſed to be different, according to what was judged moſt probable from the beſt conjecture that each could form. *Ptolemy* and his followers, as alſo *Copernicus* and *Tycho Brahe* thought it to be 1200 ſemidiameters of the Earth: *Kepler* 3500 nearly, *Ricciolus* doubles the diſtance mentioned by *Kepler*, and *Hevelius* only increaſes it by one half. But the planets Venus and Mercury having by the aſſiſtance of the teleſcope been ſeen in the diſc of the Sun, deprived of their borrowed brightneſs, it is at length found, that the apparent diameters of the planets are much leſs than they were formerly ſuppoſed; and that the ſemidiameter of Venus ſeen from the Sun ſubtends no more than a fourth part of a minute, or fifteen ſeconds, whilſt the ſemidiameter of Mercury, at its mean diſtance from the Sun, is ſeen under an angle only of ten ſeconds; that the ſemidiameter of Saturn ſeen from the Sun appears under the ſame angle, and that the ſemidiameter of Jupiter, the largeſt of all the planets, ſubtends an angle of no more than a third part of a minute at the Sun. Whence, keeping the proportion, ſome modern Aſtronomers have thought, that the ſemidiameter of the Earth, ſeen from the Sun, would ſubtend a mean angle between that larger one ſubtended by Jupiter, and that ſmaller one ſubtended by Saturn and Mercury; and equal to that ſubtended by Venus (namely fifteen ſeconds): and have thence concluded, that the Sun is diſtant from the Earth almoſt 14000 of the Earth's ſemidiameters. But the ſame authors have on another account ſomewhat increaſed this diſtance: for, inaſmuch as the Moon's diameter is a little more than a fourth part of the diameter of the Earth, if the Sun's parallax ſhould be ſuppoſed fifteen ſeconds, it would follow, that the body of the Moon is larger than that of Mercury; that is, that a ſecondary planet would be greater than a primary, which would ſeem inconſiſtent with the uniformity of the mundane ſyſtem. And on the contrary, the ſame regularity and uniformity ſeems ſcarcely to admit, that Venus, an inferior planet, and has no ſatellite, ſhould be greater than our Earth, which ſtands higher in the ſyſtem, and has ſuch a ſplendid attendant. Therefore, to obſerve a mean, let us ſuppoſe the ſemidiameter of the Earth ſeen from the Sun; or, which is the ſame thing, the Sun's horizontal parallax, to be twelve ſeconds and a half; according to which, the Moon will be leſs than Mercury, and the Earth larger than Venus; and the Sun's diſtance from the Earth will come out nearly 16500 of the Earth's ſemidiameters. This diſtance I aſſent to at preſent, as the true one, till it ſhall become certain what it is, by the experiment which I propoſe. Nor am I induced to alter my opinion by the authority of thoſe (however weighty it may be) who are for placing the Sun at an immenſe diſtance beyond the bounds here aſſigned, relying upon obſervations made upon the vibrations of a pendulum, in order to determine theſe exceeding ſmall angles; but which, as it ſeems, are not ſufficient to be depended upon: at leaſt, by this method of inveſtigating the parallax, it will come out ſometimes nothing, or even negative; that is, the diſtance would either become infinite, or greater than infinite, which is abſurd. And indeed, to confeſs the truth, it is hardly poſſible for a man to diſtinguiſh, with any degree of certainty,

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* The Royal Society.

ſeconds,

Pl. II.

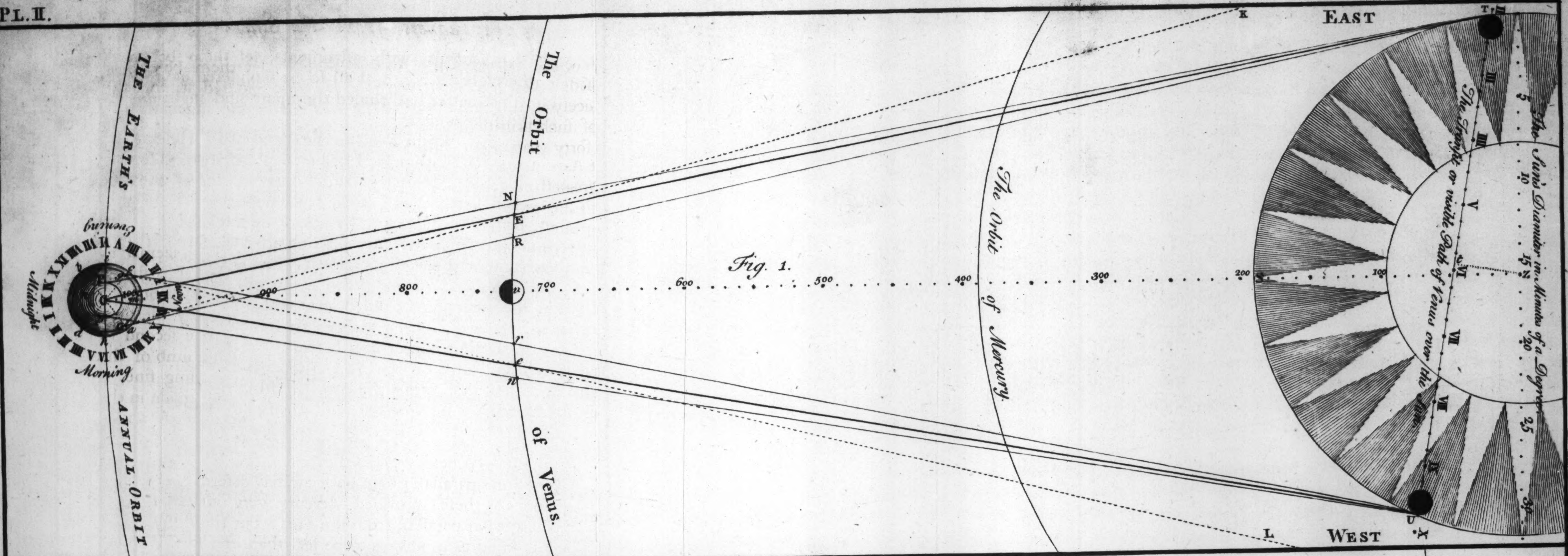
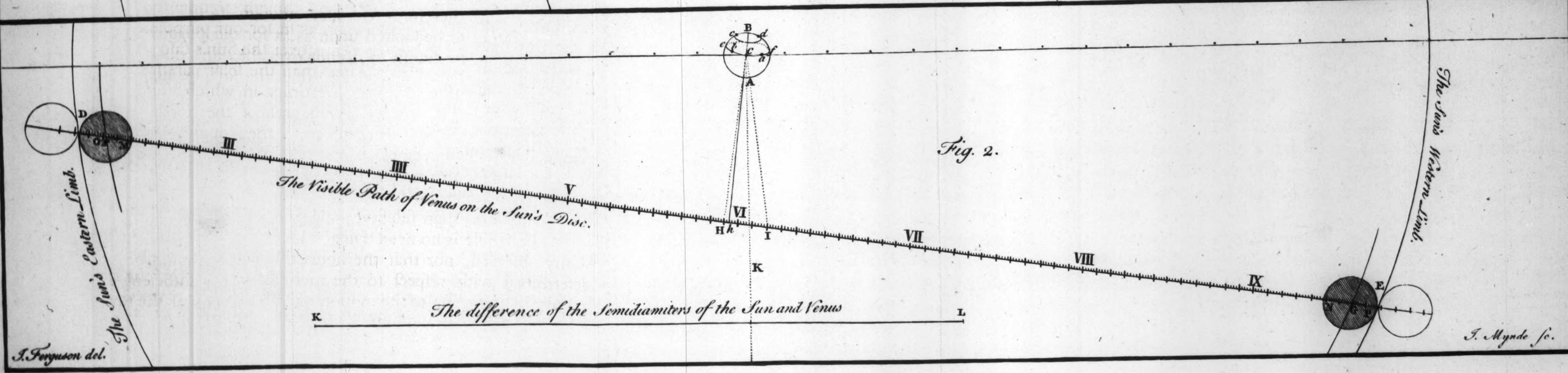


Fig. 1.


B
c
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E
III
IIII
V
VI
VII
VIII
IX
H
h
I
K
L
The Sun's Eastern Limb.
The Sun's Western Limb.
The Visible Path of Venus on the Sun's Disc.
The difference of the Semidiamiters of the Sun and Venus

Fig. 2.

J. Ferguson del.

J. Mynde sc.

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lications

ſeconds, or even ten ſeconds, with inſtruments, let them be ever ſo ſkilfully made; therefore, it is not at all to be wondered at, that the exceſſive nicety in this matter has eluded the many and ingenious endeavours of ſuch ſkilful operators.

About forty years ago, whilſt I was in the iſland of St. Helena, obſerving the ſtars about the ſouth pole, I had an opportunity of obſerving, with the greateſt diligence, Mercury paſſing over the diſc of the Sun; and (which ſucceeded better than I could have hoped for) I obſerved, with the greateſt degree of accuracy, by means of a teleſcope 24 feet long, the very moment when Mercury entering upon the Sun ſeemed to touch its limb within, and alſo the moment when going off it ſtruck the limb of the Sun's diſc, forming the angle of interior contact: whence I found the interval of time during which Mercury then appeared within the Sun's diſc, even without an error of one ſecond of time. For the lucid line intercepted between the dark limb of the planet and the bright limb of the Sun, although exceeding fine, is ſeen by the eye; and in ſtriking the eye, the little dent made in the Sun's limb, by Mercury's entering the diſc, appears to vaniſh in a moment; and alſo that made by Mercury, when leaving the diſc, ſeems to begin in an inſtant.—When I perceived this, it immediately came into my mind, that the Sun's parallax might be accurately determined by ſuch kind of obſervations as theſe; provided Mercury were but nearer to the Earth, and had a greater parallax from the Sun: but the difference of theſe parallaxes is ſo little, as always to be leſs than the ſolar parallax which we want; and therefore Mercury, though frequently to be ſeen on the Sun, is not to be looked upon as fit for our purpoſe.

There remains then the tranſit of Venus over the Sun's diſc; whoſe parallax, being almoſt four times greater than the ſolar parallax, will cauſe very ſenſible differences between the times in which Venus will ſeem to be paſſing over the Sun at different parts of the Earth. And from theſe differences, if they be obſerved as they ought, the Sun's parallax may be determined even to a ſmall part of a ſecond. Nor do we require any other inſtruments for this purpoſe, than common teleſcopes and clocks, only good of their kind; and in the obſervers, nothing more is needful than fidelity, diligence, and a moderate ſkill in Aſtronomy. For there is no need that the latitude of the place ſhould be ſcrupulouſly obſerved, nor that the hours themſelves ſhould be accurately determined with reſpect to the meridian: it is ſufficient the clocks be regulated according to the motion of the heavens, if the times be well reckoned from the total ingreſs of Venus into the Sun's diſc to the beginning of her egreſs from it; that is, when the dark globe of Venus firſt begins to touch the bright limb of the Sun; which moments, I know by my own experience, may be obſerved to within a ſecond of time.

But on account of the very ſtrict laws by which the motions of the planets are regulated, Venus is ſeldom ſeen within the Sun's diſc: and during the courſe of more than 120 years, it could not be ſeen once; namely, from the year 1639 (when this moſt pleaſing ſight happened to that excellent youth *Horrox* our countryman, and to him only, ſince the creation) to the year 1761; in which year, according to the theories which we have hitherto found agreeable to the celeſtial motions, Venus will again paſs over the Sun on the * 26th of May, in the morning; ſo that at *London*, about ſix o'clock in the morning, we may expect to ſee it near the middle of the Sun's diſc, and not above four minutes of a degree ſouth of the Sun's center. But the duration of this tranſit will be almoſt eight hours; namely, from two o'clock in the morning till almoſt ten. Hence the ingreſs will not be viſible in *England*; but as the Sun will at that time be in the 16th degree of Gemini, having almoſt 23 degrees north declination, it will be ſeen without ſetting at all in almoſt all parts of the north frigid zone: and therefore the inhabitants of the coaſt of *Norway*, beyond the city *Nidroſia*, which is called *Drontheim*, as far as the *North Cape*, will be able to obſerve Venus entering the Sun's diſc; and perhaps the ingreſs of Venus upon the Sun, when riſing, will be ſeen by the Scotch, in the northern parts of the kingdom, and by the inhabitants of the *Schetland Iſles*, formerly called *Thule*. But at the time when Venus will be neareſt the Sun's center, the Sun will be vertical to the northern ſhores of the bay of *Bengal*, or rather over the kingdom of *Pegu*; and therefore, in the adjacent regions, as the Sun, when Venus enters his diſc, will be almoſt four hours toward the eaſt, and as many to the weſt when ſhe leaves him, the apparent motion of Venus on the Sun will be accelerated by almoſt double the horizontal parallax of Venus from the Sun; becauſe Venus at that time is carried with a retrograde motion from eaſt to weſt, whilſt an eye placed upon the Earth's ſurface is whirled the contrary way, from weſt to eaſt †.

Suppoſing

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* The ſixth of June according to the New Style.

† I ſhall endeavour to explain this, and ſome of the following part of the Doctor's Eſſay, by a figure.

In Fig. 1. of Plate II. let *C* be the center of the Earth, and Z the center of the Sun. In the right line *CvZ* make *vZ* to *CZ* as 726 is to 1015 (§ 37). Let *abcd* be the Earth, *v* Venus's place in her orbit at the time of her conjunction with the Sun; and let *TSV* be the Sun, whoſe diameter is 31′ 42″.

The motion of Venus in her orbit is in the direction *Nvn*, and the Earth's motion on its axis is according to the order of the 24 hours placed around it in the figure. Therefore, ſuppoſing the mouth of the *Ganges* to be at *G*, when Venus is at *E* in her orbit, and

to

Suppoſing the Sun's parallax (as we have ſaid) to be $12\frac{1}{2}''$, the parallax of Venus will be $43''$; from which ſubtracting the parallax of the Sun, there will remain $30''$ at leaſt for the horizontal parallax of Venus from the Sun; and therefore the motion of Venus will be increaſed

to be carried from *G* to *g* by the Earth's motion on its axis, whilſt Venus moves from *E* to *e* in her orbit; it is plain that the motions of Venus and the *Ganges* are contrary to each other.

The true motion of Venus in her orbit, and conſequently the ſpace ſhe ſeems to run over on the Sun's diſc in any given time, could be ſeen only from the Earth's center *C*, which is at reſt. And as ſeen from that point, her tranſit over the Sun would be in the right line *TtU*; and her motion therein at the rate of four minutes of a degree *per* hour. *T* is the point of the Sun's eaſtern limb which Venus would ſeem to touch at the moment of her total ingreſs on the Sun, as ſeen from *C*, when ſhe is at *E* in her orbit; and *U* is the point of the Sun's weſtern limb which ſhe would ſeem to touch at the moment of her beginning of egreſs from the Sun, as ſeen from *C*, when ſhe is at *e* in her orbit.

When the *Ganges* is at *m* (in revolving through the arc *Gmg*) the Sun is on its meridian. Therefore, ſince *G* and *g* are equally diſtant from *m* at the beginning and ending of the tranſit, it is plain that the Sun will be as far eaſt of the meridian of the *Ganges* (at *G*) when the tranſit begins, as it will be weſt of the meridian of the ſame place (revolved from *G* to *g*) when the tranſit ends.

But although the beginning of the tranſit, or rather the moment of Venus's total ingreſs upon the Sun at *T*, as ſeen from the Earth's center, muſt be when Venus is at *E* in her orbit, becauſe ſhe is then ſeen in the direction of the right line *CET*; yet, at the ſame inſtant of time, as ſeen from the *Ganges* at *G*, ſhe will be ſhort of her ingreſs on the Sun, being then ſeen eaſtward of him, in the right line *GEK*, which makes the angle *KET* (equal to the oppoſite angle *GEC*) with the right line *CET*. This angle is called the angle of Venus's parallax from the Sun, which retards the beginning of the tranſit as ſeen from the banks of the *Ganges*; ſo that the *Ganges* muſt advance a little farther toward *m*, and Venus muſt move on in her orbit from *E* to *R*, before ſhe can be ſeen from *G* (in the right line *GRT*) wholly within the Sun's diſc at *T*.

Again, when Venus comes to *e* in her orbit, ſhe will be ſeen at *U* from the Earth's center *C*, juſt beginning to leave the Sun, that is, at the beginning of her egreſs from his weſtern limb: but at the ſame inſtant of time, as ſeen from the *Ganges* which is then at *g*, ſhe will be quite clear of the Sun toward the weſt; being then ſeen from *g* in the right line *geL*, which makes an angle, as *UeL* (equal to the oppoſite angle *Ceg*) with the right line *CeU*: and this is the angle of Venus's parallax from the Sun as ſeen from the *Ganges* at *g*, when ſhe is but juſt beginning to leave his diſc at *U*, as ſeen from the Earth's center *C*.

Here it is plain, that the duration of the tranſit about the mouth of the *Ganges* (and alſo in the neighbouring places) will be diminiſhed by double the quantity of Venus's parallax from the Sun at the beginning or ending of the tranſit. For Venus muſt be at *E* in her orbit when ſhe is wholly upon the Sun at *T*, as ſeen from the Earth's center *C*: but at that time ſhe is ſhort of the Sun as ſeen from the *Ganges* at *G*, by the whole quantity of her eaſtern parallax from the Sun at that time, which is the angle *KET*. [This angle, in fact, is only $23''$; though it is repreſented to be much larger in the figure, becauſe the Earth therein is a vaſt deal too big.] Now, as Venus moves at the rate of $4'$ *per* hour, ſhe will move $23''$ in 5 minutes 45 ſeconds: and therefore, the tranſit will be 5 min. 45 ſeconds later of beginning at the banks of the *Ganges* than at the Earth's center.——When the tranſit is ending at *U*, as ſeen from the Earth's center *C*, Venus will be quite clear of the Sun (by the whole quantity of her weſtern parallax from him) as ſeen creaſed $45''$ at leaſt by that parallax, whilſt ſhe paſſes over the Sun's diſc, in thoſe elevations of the pole which are in places near the tropic, and yet more in the neighbourhood of the equator. Now, Venus at that time will move on the Sun's diſc very nearly at the rate of four minutes of a degree in an hour; and therefore 11 minutes of time at leaſt are to be allowed for $45''$ or three fourths of a minute of a degree; and by this ſpace of time, the duration of this eclipſe cauſed by Venus will, on account of the parallax, be ſhortened. And from this ſhortening of the time only, we might ſafely enough draw a concluſion concerning the parallax which we are in ſearch of, provided the diameter of the Sun, and the latitude of Venus, were accurately known. But we cannot expect an exact computation in a matter of ſuch ſubtilty.

We muſt endeavour therefore to obtain, if poſſible, another obſervation, to be taken in thoſe places where Venus will be in the middle of the Sun's diſc at midnight; that is, in places under the oppoſite meridian to the former, or about 6 hours or 90 degrees weſt of London; and where Venus enters upon the Sun a little before its ſetting, and goes off a little after its riſing. And this will happen under the abovementioned meridian, and where the elevation of the north pole is about 56 degrees; that is, in a part of Hudſon's Bay, near a place called *Port-Nelſon*. For, in this and the adjacent places, the parallax of Venus will increaſe the duration of the tranſit by at leaſt ſix minutes of time; becauſe, whilſt the Sun, from its ſetting to its riſing, ſeems to paſs under the pole, thoſe places on the Earth's diſc will be carried with a motion from eaſt to weſt, contrary to the motion of the *Ganges)*; that is, with a motion conſpiring with the motion of Venus; and therefore, Venus will ſeem to move more ſlowly on the Sun, and to be longer in paſſing over his diſc*.

If

ſeen from the *Ganges*, then at *g*: and this parallax will be $22''$, equal to the ſpace through which Venus moves in 5 minutes 30 ſeconds of time: ſo that the tranſit will end $5\frac{1}{2}$ minutes ſooner as ſeen from the *Ganges*, than as ſeen from the Earth's center.

Hence, the whole contraction of the duration of the tranſit at the mouth of the *Ganges* will be 11 minutes 15 ſeconds of time: for it is 5 minutes 45 ſeconds at the beginning, and 5 minutes 30 ſeconds at the end.

* In Fig. 1. of Plate II. let *aC* be the meridian of the eaſtern mouth of the *Ganges*, and *bC* the meridian of *Port-Nelſon* at the mouth of *York-River* in *Hudſon's Bay*, 56° north latitude. As the meridian of the *Ganges* revolves from *a* to *c*, the meridian of *Port-Nelſon* will revolve from *b* to *d*: therefore, whilſt the *Ganges* revolves from *G* to *g*, through the arc *Gmg*, *Port-Nelſon* revolves the contrary way (as ſeen from the Sun or Venus) from *P* to *p*, through the arc *Pnp*.——Now, as the motion of Venus is from *E* to *e* in her orbit, whilſt ſhe ſeems to paſs over the Sun's diſc in the right line *Tt*[illegible] as ſeen

If therefore it ſhould happen that this tranſit ſhould be properly obſerved by ſkilful perſons at both theſe places, it is clear, that the duration thereof will be 17 minutes longer, as ſeen from *Port-Nelſon*, than as ſeen from the *Eaſt-Indies*. Nor is it of much conſequence (if the Engliſh ſhall at that time give any attention to this affair) whether the obſervation be made at *Fort-George*, commonly called *Madras*, or at *Bencoolen* on the weſtern ſhore of the iſland of *Sumatra*, near the equator. But if the French ſhall be diſpoſed to take any pains herein, an obſerver may ſtation himſelf conveniently enough at *Pondichery* on the weſt ſhore of the bay of *Bengal*, where the altitude of the pole is about 12 degrees. As to the Dutch, their celebrated mart at *Batavia* will afford them a place of obſervation fit enough for this purpoſe, provided they alſo have but a diſpoſition to aſſiſt in advancing, in this particular, the knowledge of the heavens.——And indeed I could wiſh that many obſervations of the ſame phenomenon might be taken by different perſons at ſeveral places, both that we might arrive at a greater degree of certainty by their agreement, and alſo leſt any ſingle obſerver ſhould be deprived, by the intervention of clouds, of a ſight, which I know not whether any man living in this or the next age will ever ſee again; and on which depends the certain and adequate ſolution of a problem the moſt noble, and at any other time not to be attained to. I recommend it therefore, again and again, to thoſe curious Aſtronomers, who (when I am dead) will have an opportunity of obſerving theſe things, that they would remember this my admonition, and diligently apply themſelves with all their might to the making this obſervation:

ſeen from the Earth's center *C*, it is plain, that whilſt the motion of the *Ganges* is contrary to the motion of Venus in her orbit, and thereby ſhortens the duration of the tranſit at that place, the motion of *Port-Nelſon* is the ſame way as the motion of Venus, and will therefore increaſe the duration of the tranſit: which may in ſome degree be illuſtrated by ſuppoſing, that whilſt a ſhip is under ſail, if two birds fly along the ſide of the ſhip in contrary directions to each other, the bird which flies contrary to the motion of the ſhip will paſs by its whole ſide ſooner than the other bird will, which flies the ſame way as the ſhip ſails.

In fine, it is plain by the figure, that the duration of the tranſit muſt be longer as ſeen from *Port-Nelſon*, than as ſeen from the Earth's center, and longer as ſeen from the Earth's center, than as ſeen from the mouth of the *Ganges*.——For *Port-Nelſon* muſt be at *P*, and Venus at *N* in her orbit, when ſhe appears wholly within the Sun at *T*; and the ſame place muſt be at *p*, and Venus at *n*, when ſhe appears at *U*, beginning to leave the Sun.—The *Ganges* muſt be at *G*, and Venus at *R*, when ſhe appears from *G* upon the Sun at *T*; and the ſame place muſt be at *g*, and Venus at *r*, when ſhe appears from *g* beginning to leave the Sun at *U*. So that Venus muſt move from *N* to *n* in her orbit, whilſt ſhe is ſeen to paſs over the Sun from *Port-Nelſon*; from *E* to *e* in paſſing over the Sun, as ſeen from the Earth's center; and only from *R* to *r* whilſt ſhe paſſes over the Sun, as ſeen from the banks of the *Ganges*.

tion: and I earneſtly wiſh them all imaginable ſucceſs; in the firſt place that they may not, by the unſeaſonable obſcurity of a cloudy ſky, be deprived of this moſt deſirable ſight; and then, that having aſcertained with more exactneſs the magnitudes of the planetary orbits, it may redound to their immortal fame and glory.

We have now ſhewn, that by this method the Sun's parallax may be inveſtigated to within its five hundredth part, which doubtleſs will appear wonderful to ſome. But if an accurate obſervation be made in each of the places above marked out, we have already demonſtrated that the durations of this eclipſe made by Venus will differ from each other by 17 minutes of time; that is, upon a ſuppoſition that the Sun's parallax is $12\frac{1}{2}''$. But if the difference ſhall be found by obſervation to be greater or leſs, the Sun's parallax will be greater or leſs, nearly in the ſame proportion. And ſince 17 minutes of time are anſwerable to $12\frac{1}{2}$ ſeconds of ſolar parallax, for every ſecond of parallax there will ariſe a difference of more than 80 ſeconds of time: whence, if we have this difference true to two ſeconds, it will be certain what the Sun's parallax is, to within a 40th part of one ſecond; and therefore, his diſtance will be determined to within its 500dth part at leaſt, if the parallax be not found leſs than what we have ſuppoſed: for 40 times $12\frac{1}{2}$ make 500.

And now I think I have explained this matter fully, and even more than I needed to have done, to thoſe who underſtand Aſtronomy: and I would have them take notice, that on this occaſion, I have had no regard to the latitude of Venus, both to avoid the inconvenience of a more intricate calculation, which would render the concluſion leſs evident; and alſo becauſe the motion of the nodes of Venus is not yet diſcovered, nor can be determined but by ſuch conjunctions of the planet with the Sun as this. For we conclude, that Venus will paſs 4 minutes below the Sun's center, only in conſequence of the ſuppoſition that the plane of Venus's orbit is immovable in the ſphere of the fixed ſtars, and that its nodes remain in the ſame places where they were found in the year 1639. But if Venus, in the year 1761, ſhould move over the Sun in a path more to the ſouth, it will be manifeſt that her nodes have moved backward among the fixed ſtars; and if more to the north, that they have moved forward; and that at the rate of $5\frac{1}{2}$ minutes of a degree in 100 Julian years, for every minute that Venus's path ſhall be more or leſs diſtant than the aboveſaid 4 minutes from the Sun's center. And the difference between the durations of theſe eclipſes will be ſomewhat leſs than 17 minutes of time, on account of Venus's ſouth latitude; but greater, if by the motion of the nodes forward ſhe ſhould paſs on the north of the Sun's center.

But for the ſake of thoſe, who though they are delighted with ſidereal obſervations, may not yet have made themſelves acquainted with the doctrine of parallaxes, I choſe to explain the thing a little more fully by a ſcheme, and alſo by a calculation ſomewhat more accurate.

Let us ſuppoſe that at London, in the year 1761, on the 6th of June, at 55 minutes after V in the morning, the Sun will be in Gemini 15° 37′, and therefore that at its center the ecliptic is inclined toward the north, in an angle of 6° 10′; and that the viſible path of Venus on the Sun's diſc at that time declines to the ſouth, making an angle with the ecliptic of 8° 28′: then the path of Venus will alſo be inclined to the ſouth, with reſpect to the equator, interſecting the parallels of declination at an angle of 2° 18′ *. Let us alſo ſuppoſe, that Venus, at the forementioned time, will be at her leaſt diſtance from the Sun's center, *viz.* only four minutes to the ſouth; and that every hour ſhe will deſcribe a ſpace of 4 minutes on the Sun, with a retrograde motion. The Sun's ſemidiameter will be 15′ 51″ nearly, and that of Venus 37½″. And let us ſuppoſe, for trial's ſake, that the difference of the horizontal parallaxes of Venus and the Sun (which we want) is 31″, ſuch as it comes out if the Sun's parallax be ſuppoſed 12½″. Then, on the center *c* (Plate II. Fig. 2.) let the little circle *AB*, repreſenting the Earth's diſc, be deſcribed, and let its ſemidiameter *cB* be 31″; and let the elliptic parallels of 22 and 56 degrees of north latitude (for the *Ganges* and *Port-Nelſon)* be drawn within it, in the manner now uſed by Aſtronomers for conſtructing ſolar eclipſes. Let *BcAb* be the meridian in which the Sun is, and to this, let the right line *FHG*, repreſenting the path of Venus, be inclined at an angle of 2° 18′; and let it be diſtant from the center *c* 240 ſuch parts, whereof *cB* is 31. From *c* let fall the right line *cH*, perpendicular to *FG*; and ſuppoſe Venus to be at *H* at 55 minutes after V in the morning. Let the right line *FGH* be divided into the horary ſpaces III IV, IV V, V VI, *&c.* each equal to *cH*; that is, to 4 minutes of a degree. Alſo, let the right line *KL* be equal to the difference of the apparent ſemidiameters of the Sun and Venus, which is 15′ 13½″; and a circle being deſcribed with the radius *KL*, on a center taken in any point within the little circle *AB* repreſenting the Earth's diſc, will meet the right line *FG* in a point denoting the time at London when Venus ſhall touch the Sun's limb internally, as ſeen from the place of the Earth's ſurface that anſwers to the point aſſumed in the Earth's diſc. And if a circle be deſcribed on the center *c*, with the radius *KL*, it will meet the right line *FG* in the points *F* and *G*; and the ſpaces *FH* and *GH* will be each equal to 14′ 4″, which ſpace Venus will appear to paſs over in 3 hours 40 minutes of time at London; therefore, *F* will fall in II hours 15 minutes, and *G* in IX hours 35 minutes in the morning. Whence it is manifeſt, that if the magnitude of the Earth, on account of its immenſe diſtance, ſhould vaniſh as it were into a point; or if, being deprived of a diurnal motion, it ſhould always have the Sun vertical to the ſame point *c*, the whole duration of this eclipſe would be 7 hours 20 minutes. But the Earth in that time being whirled through 110 degrees of longitude, with a motion contrary to the motion of Venus, and conſequently the abovementioned duration being contracted, ſuppoſe 12 minutes, it will come out 7 hours 8 minutes, or 107 degrees, nearly.

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* I am fully convinced that this is an overſight in the Doctor, occaſioned by his placing both the Earth's axis *cb* (Fig. 2. of Plate II.) and the axis of Venus's orbit *cH* on the ſame ſide of the axis of the ecliptic *cK*; the former making an angle of 8° 28′ therewith, and the latter an angle of 6° 10′; the difference of which angles is only 2° 18′. But the truth is, that the Earth's axis, and the axis of Venus's orbit will lie on different ſides of the axis of the ecliptic, the former making an angle of 6° therewith, and the latter an angle of 8½. Therefore, the ſum of theſe angles, which is 14°½ (and not their difference 2° 18′) is the inclination of Venus's viſible path to the equator and parallels of declination.

Now, Venus will be at *H*, at her leaſt diſtance from the Sun's center, when in the meridian of the eaſtern mouth of the *Ganges*, where the altitude of the pole is about 22 degrees. The Sun therefore will be equally diſtant from the meridian of that place, at the moments of the ingreſs and egreſs of the planet, *viz.* 53½ degrees; as the points *a* and *b* (repreſenting that place in the Earth's diſc *AB*) are, in the greater parallel, from the meridian *BcA*. But the diameter *ef* of that parallel will be to the diſtance *ab*, as the ſquare of the radius to the rectangle under the ſines of 53½ and 68 degrees: that is, as 1′ 2″ to 46″ 13‴. And by a good calculation, (which, that I may not tire the reader, it is better to omit) I find, that the circle deſcribed on the center *a*, with the radius *KL*, will meet the right line *FH* in the point *M*, at II hours 20 minutes 40 ſeconds; but that being deſcribed round *b* as a center, it will meet *HG* in the point *N* at IX hours 29 minutes 22 ſeconds, according to the time reckoned at London: and therefore, Venus will be ſeen entirely within the Sun at the banks of the *Ganges* for 7 hours 8 minutes 42 ſeconds: we have then rightly ſuppoſed that the duration will be 7 hours 8 minutes, ſince the part of a minute is here of no conſequence.

But adapting the calculation to *Port-Nelſon*, I find, that the Sun being about to ſet, Venus will enter his diſc; and immediately after his riſing ſhe will leave the ſame. That place is carried in the intermediate time through the hemiſphere oppoſite to the Sun, from *c* to *d*, with a motion conſpiring with the motion of Venus; and therefore, the ſtay of Venus on the Sun will be about 4 minutes longer, on account of the parallax; ſo that it will be at leaſt 7 hours 24 minutes, or 111 degrees of the equator. And ſince the latitude of the place is 56 degrees, as the ſquare of the radius is to the rectangle contained under the ſines of $55\frac{1}{2}$ and 34 degrees, ſo is *AB* which is 1′ 2″, to *cd* which is 28″ 33‴. And if the calculation be juſtly made, it will appear, that a circle deſcribed on the center *a* with the radius *KL*, will meet the right line *FH* in *O*, at II hours 12 minutes 45 ſeconds; and that ſuch a circle deſcribed on *d* as a center will meet *HG* in *P*, at IX hours 36 minutes 37 ſeconds; and therefore the duration at *Port-Nelſon* will be 7 hours 23 minutes 52 ſeconds, which is greater than at the mouth of the *Ganges*, by 15 minutes 10 ſeconds of time. But if Venus ſhould paſs over the Sun without having any latitude, the difference would be 18 minutes 40 ſeconds; and if ſhe ſhould paſs 4′ north of the Sun's center, the difference would amount to 21 minutes 40 ſeconds, and will be ſtill greater, if the planet's north latitude be more increaſed.

From the foregoing hypotheſis it follows, that at London, when the Sun riſes, Venus will have entered his diſc; and that, at IX hours 37 minutes in the morning ſhe will touch the limb of the Sun internally in going off; and laſtly, that ſhe will not entirely leave the Sun till IX hours 56 minutes.

It likewiſe follows from the ſame hypotheſis, that the center of Venus ſhould juſt touch the Sun's northern limb in the year 1769, on the third of June at XI o'clock at night. So that, on account of the parallax, it will appear in the northern parts of *Norway* entirely within the Sun, which then does not ſet to thoſe parts; whilſt, on the coaſts of *Peru* and *Chili* it will ſeem to travel over a ſmall portion of the diſc of the ſetting Sun; and over that of the riſing Sun at the *Molucca Iſlands*, and in their neighbourhood.—But if the nodes of Venus be found to have a retrograde motion (as there is reaſon to believe from ſome later obſervations they have) then Venus will be ſeen every where within the Sun's diſc; and will afford a much better method for finding the Sun's parallax, by almoſt the greateſt difference in the durations of theſe eclipſes that can poſſibly happen.

But how this parallax may be deduced from obſervations made ſomewhere in the *Eaſt-Indies*, in the year 1761, both of the ingreſs and egreſs of Venus, and compared with thoſe made in its going off with us; namely, by applying the angles of a triangle given in ſpecie to the circumference of three equal circles; ſhall be explained on ſome other occaſion.

ARTICLE VII.

Shewing that the Sun's parallax cannot be ſo nearly aſcertained by the tranſit of Venus, as has been generally expected; and why.

50. Although it is evident that Doctor HALLEY has explained his method with great modeſty and ſome doubtfulneſs, and told us that the Sun's parallax may be only determined within its five hundredth part thereby, provided it be not leſs than $12\frac{1}{2}''$, and that Venus ſhould not paſs more than 4 minutes of a degree below the Sun's center, which he even ſuſpects ſhe will, and has taken all proper pains not to raiſe our expectations too high on that account; yet, from his well known abilities, and character as a great Aſtronomer, it ſeems mankind in general have laid greater ſtreſs upon the method he has preſcribed, than he ever deſired them to do. Only, as he was well convinced it was by far the beſt method by which this high problem can ever be ſolved, he warmly recommended it on that account.—It is plain that he had not then made a ſufficient number of obſervations, whereby to determine with certainty whether the nodes of Venus's orbit had any motion at all; or if they had, whether it be forward or backward, with reſpect to the fixed ſtars. And conſequently, having not then made his own tables, he was obliged to take his calculations from others, which allow of no motion to theſe nodes, and had alſo reckoned the time of the tranſit to be about half an hour too late.

51. But more modern obſervations prove, and ſo do the Doctor's tables which were made from his own obſervations after he wrote the above paper, that the nodes of Venus's orbit move backward in the ſtarry heaven, at ſuch a rate, that Venus will be almoſt 10 minutes of a degree below the Sun's center at the middle of her tranſit; by which means, the line of the tranſit will be ſo much ſhortened, as will make the whole duration of Venus on the Sun to be about an hour and a quarter leſs than if ſhe had paſſed only 4 minutes below the center

of

of the Sun; and therefore, her parallax from the Sun will be ſo much diminiſhed at places on the Earth's ſurface, both at the beginning and end of the tranſit, that the difference of its durations as ſeen from the banks of the *Ganges*, or from *Pondichery*, *Bencoolen*, or *Batavia*, will not amount to 11 minutes of time, from that of its duration as ſeen from the Earth's center.

52. But this is not all: for although the tranſit will begin at *Port-Nelſon* before the Sun ſets to that place, it will be entirely over before he riſes next morning, on account of its ending ſo much ſooner than the Doctor expected from the tables, to which he was then obliged to truſt. So that, as no purpoſe could be anſwered by ſending obſervers to *Port-Nelſon*, none will be ſent to that place; and hence we are quite deprived of the advantage that was judged would ariſe from the contrary motion of *Port-Nelſon* to that of the *Ganges* and adjacent places in the *Eaſt-Indies*.

53. As I now intend to trace this affair through all its intricacies, ſo as to render it as intelligible to the reader as I can, there will be an unavoidable neceſſity of dwelling much longer upon it than I could wiſh. And as it is impoſſible to lay down truly the parallels of latitude of places, and the ſituations of the places themſelves, in ſuch a ſmall diſc of the Earth as the Doctor gives in his diagram, ſo as thereby to meaſure the durations of the tranſit, with the times of its beginning and ending at theſe places, and for which he confeſſes that he was obliged to make tedious calculations; I ſhall take another method, whereby the quantities of the parallaxes may be eaſily meaſured on a ſcale with a pair of compaſſes; and then ſhew by a very eaſy calculation in the common rule of proportion, how much the duration of the tranſit, at any given place, is affected by the parallax.—In this, I ſhall firſt ſuppoſe with the Doctor that the Sun's parallax is $12\frac{1}{2}''$, and then ſhew, in the next article, how nearly its real quantity may be found, from the obſerved difference of the duration of the tranſit at different places.

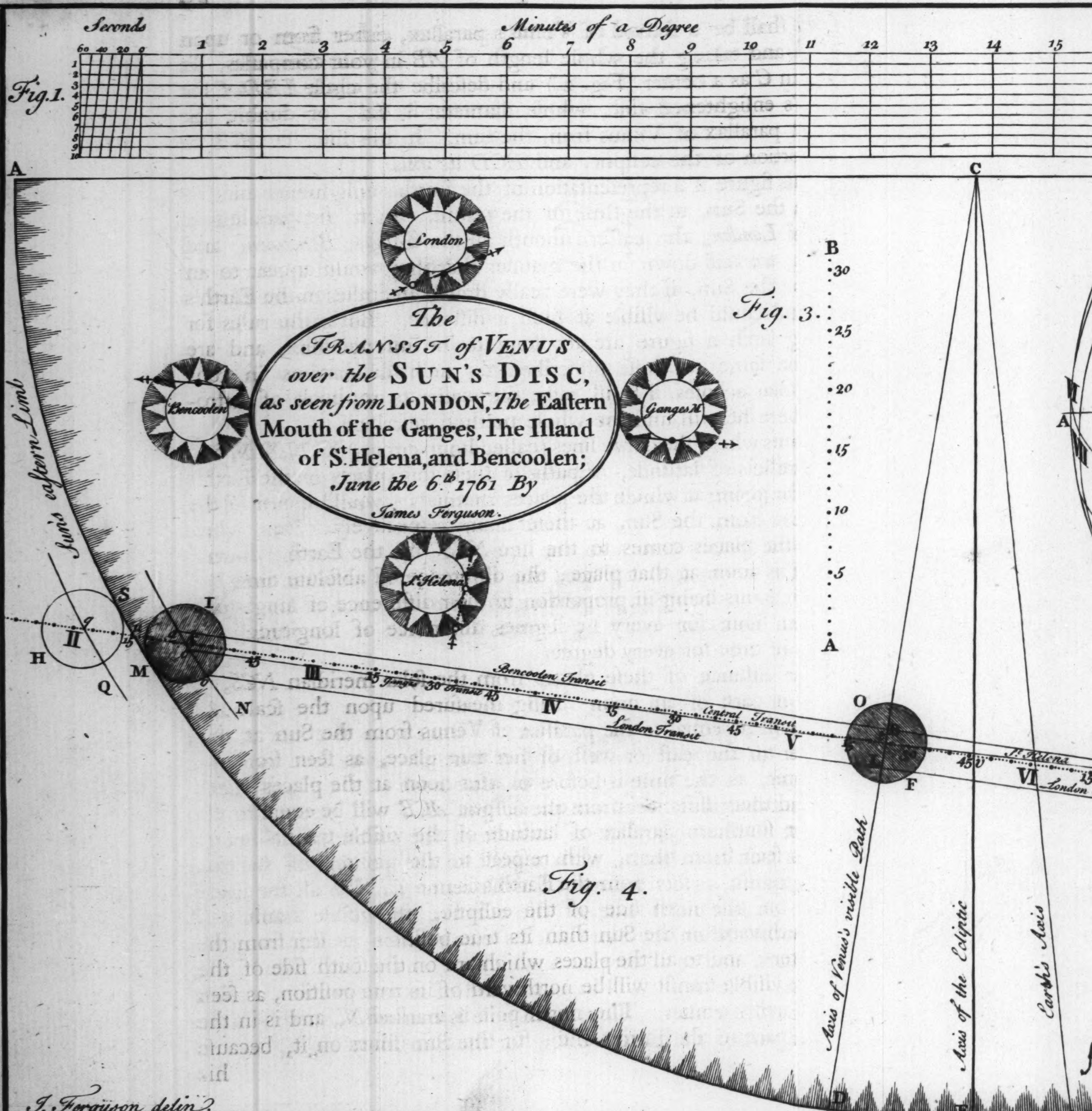
54. Both the Sun's center and the Earth's are always in the plane of the ecliptic. But as the north pole will incline $22\frac{2}{3}$ degrees toward the Sun on the day of the tranſit, becauſe the Sun's declination will then be $22\frac{2}{3}$ degrees north of the equator, all thoſe northern places whoſe latitude is greater than the Sun's declination, and which are on the enlightened part of the Earth during the time of the tranſit, will be on the northern ſide of the plane of the ecliptic; and therefore, the tranſit-line will be farther ſouth on the Sun's diſc, as ſeen from them, than as ſeen from the Earth's center: and the farther ſouth the ſaid line is on the Sun, the ſhorter it will be; and ſo of courſe will the duration of

the tranſit. Thoſe places whoſe northern latitude is leſs than the Sun's declination, and are on the enlightened part of the Earth during the tranſit (eſpecially if its middle be about their noon) will be on the ſouth ſide of the plane of the ecliptic; and ſo alſo muſt all the places be which lie on the ſouth ſide of the equator; and therefore, to all theſe, the viſible line of the tranſit will be more northward upon the Sun, than it would be as ſeen from the Earth's center, and the duration will thereby be increaſed.—But to thoſe places which move in the plane of the ecliptic at that time, the line of the tranſit, as ſeen from them, will be the ſame upon the Sun, as if it were ſeen from the Earth's center; and ſo the duration of the tranſit at them can be affected by nothing beſides the eaſtern or weſtern parallax of Venus from the Sun, at the time when the tranſit begins or ends at the Earth's center. And hence it may be preſum'd, that ſome places in the neighbourhood of the *Ganges*, (for example, *Calcutta)* would be very proper for obſervation, ſince they would have the whole advantage of the parallax of longitude to contract the duration, without almoſt any parallax of latitude either to contract or to lengthen it. For, both at *London* and *Bencoolen*, theſe parallaxes act againſt one another: thus, at London, the parallax of longitude (or of Venus from the Sun) increaſes the duration of the tranſit at its ending; but the parallax of latitude, ariſing from the northern ſituation of London, diminiſhes it, and that in ſuch a manner, as their effects will nearly deſtroy one another: and at *Bencoolen*, *Batavia*, and the adjacent places, the parallax of longitude diminiſhes the duration of the tranſit, and the parallax of latitude increaſes it; but it will be more ſhortened by the former, than lengthened by the latter.—What allowances muſt be made for the affections by parallax of latitude, we ſhall ſhew in the next article.

55. It is plain by § 38 and 39, that if the Sun's horizontal parallax be $12\frac{1}{2}''$, the horizontal parallax of Venus from the Sun, on the day of the tranſit, muſt be $31''$. Therefore, to thoſe places near the equator, where the Sun riſes at the time the tranſit begins as ſeen from the Earth's center, Venus will be $31''$ ſhort of the Sun toward the eaſt; and to thoſe places about the equator where the Sun ſets at the end of the tranſit, ſhe will be $31''$ clear of the Sun to the weſt. The Sun will be juſt riſen on the north-eaſt coaſt of Africa; in the kingdom of *Adel*, when the tranſit begins; and will ſet in *New Guinea*, eaſt of the iſle of *Celebes*, when it ends.

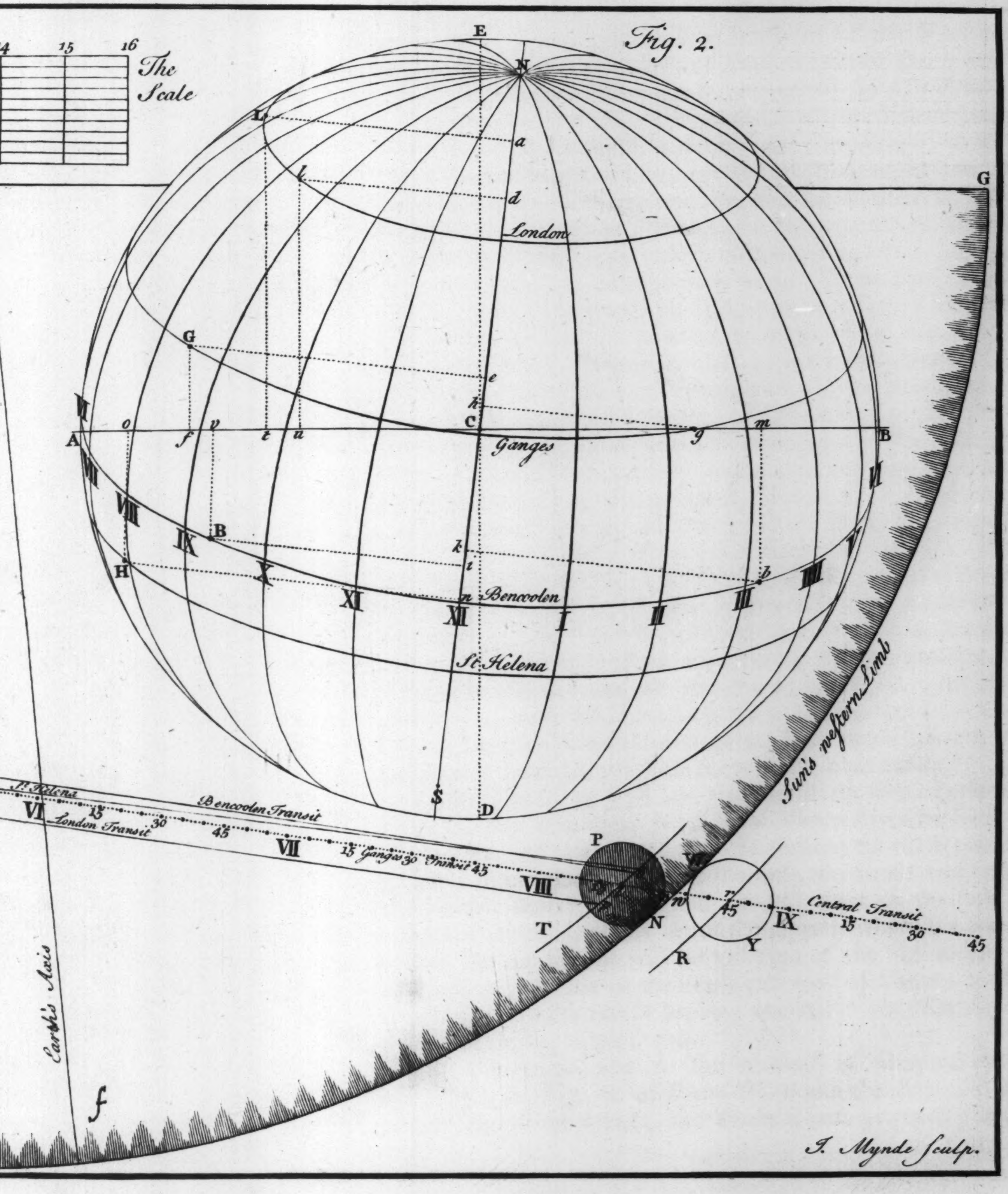
56. We come now to the method of conſtruction mentioned in § 53. In Fig. 3. of Plate III. make the line *AB* of any convenient length for a ſcale, and divide it into 31 equal parts, every one of

PL. III.



Seconds
60 40 20 0
Minutes of a Degree
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Fig. 1.
1 2 3 4 5 6 7 8 9 10
A
C
B
30
25
20
15
10
5
A
Fig. 3.
London
Bencoolen
Ganges M.
St. Helena
The TRANSIT of VENUS over the SUN's DISC, as seen from LONDON, The Eastern Mouth of the Ganges, The Island of St. Helena, and Bencoolen; June the 6th 1761 By James Ferguson.
Sun's eastern Limb
S
I
II
H
M
Q
N
III
IV
V
VI
Bencoolen Transit
Ganges 30 Transit 45
Central Transit
London Transit
30
45
15
45
O
F
45 V
St. Helena
London
Fig. 4.
Axis of Venus's visible Path
Axis of the Ecliptic
Earth's Axis
D
E
J. Ferguson delin.

Fig. 2.



J. Mynde sculp.

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of which ſhall be a ſecond of Venus's parallax, either from or upon the Sun: and taking the whole length of *AB* in your compaſſes, ſet one foot in *C* as a center (Fig. 2.) and deſcribe the circle *EBDA* for the Earth's enlightened diſc, whoſe diameter is 62″, or double the horizontal parallax of Venus from the Sun. In this diſc, let *ACB* be a ſmall portion of the ecliptic, and *ECD* its axis.

57. This figure is a repreſentation of the Earth's enlightened diſc, as ſeen from the Sun, at the time of the tranſit. In it, the parallels of latitude of *London*, the eaſtern mouth of the *Ganges*, *Bencoolen*, and *St. Helena*, are laid down in the manner that they would appear to an obſerver on the Sun, if they were really drawn in circles on the Earth's ſurface, and could be viſible at ſuch a diſtance. But as the rules for delineating ſuch a figure are too long to be inſerted here, and are exactly the ſame as thoſe preſcribed for ſuch delineations in conſtructing ſolar eclipſes, I muſt refer the reader to my book of Aſtronomy, where he will find the whole method deſcribed.

The points where the curve lines (called hour-circles) XI *N*, X *N*, &c. cut the parallels of latitude, or paths of theſe four places on the Earth's diſc, are the points at which the places themſelves would appear in the diſc, as ſeen from the Sun, at theſe hours reſpectively. And when either of the places comes to the line *NCS*, by the Earth's diurnal motion, it is noon at that place; the difference of abſolute time between their noons being in proportion to their difference of longitudes, allowing an hour for every 15 degrees difference of longitude, and 4 minutes of time for every degree.

58. The diſtance of theſe places from the ſolar meridian *NCS*, at any hour or part of an hour, being meaſured upon the ſcale *AB* (Fig. 3.) will be equal to the parallax of Venus from the Sun at that time, either to the eaſt or weſt of her true place, as ſeen from the Earth's center, as the time is before or after noon at the places themſelves. And their diſtances from the ecliptic *ACB* will be equal to the northern or ſouthern parallax of latitude of the viſible tranſit-line on the Sun, as ſeen from them, with reſpect to the poſition of the true line of the tranſit, as ſeen from the Earth's center *C*. To all the places which are on the north ſide of the ecliptic, the viſible tranſit will be more ſouthward on the Sun than its true poſition, as ſeen from the Earth's center; and to all the places which are on the ſouth ſide of the ecliptic, the viſible tranſit will be northward of its true poſition, as ſeen from the Earth's center. The north pole is marked *N*, and is in the enlightened part of the Earth's diſc; for the Sun ſhines on it, becauſe

his declination is north: and the ſouth pole is marked *S*, which lies hid in the obſcure part of the diſc.

59. The true duration of the tranſit could be ſeen only by an obſerver at the Earth's center (ſuppoſing the Earth to be tranſparent like glaſs) becauſe his place would have no motion, and conſequently there could be no parallax to affect the duration. To ſuch an obſerver, the whole duration, from the moment of Venus's total ingreſs upon the Sun to the beginning of her egreſs from him, would be 5 hours 52 minutes, as will evidently appear by a projection of the tranſit-line on the Sun's diſc, croſſing the axis of the ecliptic 9′ 43″ below the Sun's center; in which line, the true motion of Venus, as ſeen from the Earth's center, is at the rate of 4 minutes of a degree *per* hour, according to the tables; and the points in that line which are hid by the center of Venus, at the moments of her two internal contacts with the Sun's limb, are 28 minutes paſt II in the morning at London, and 20 minutes paſt VIII.

60. We ſhall reckon the whole duration of the tranſit to be the interval between the moments of the two internal contacts of Venus with the Sun; that is, the moment of her total ingreſs upon the Sun ſhall be called the beginning of the tranſit, and the moment when her egreſs from the Sun begins ſhall be called the end; becauſe theſe inſtants are the moſt conſpicuous for obſervation; and Dr. HALLEY has aſſured us that they may be obſerved, each, to a ſingle ſecond of time; ſince he could obſerve the like in the tranſit of Mercury, although that planet is much farther from the Earth than Venus is, at ſuch tranſits.

61. The beginning of the tranſit will not be viſible at London, becauſe the Sun will not then be riſen; but the middle and end will be ſeen.—At the middle, *London* will be at *L* on the Earth's diſc, as ſeen from the Sun, and at *l* when it ends. *See Fig.* 2.

Draw *La* and *ld*, each perpendicular to the general meridian *NCS*; and taking *La* in your compaſſes, meaſure that extent on the ſcale *AB* (Fig. 3.) and you will find it to be 19 ſeconds, which is the eaſtern parallax of Venus on the Sun, as ſeen from *London*, at the middle of the tranſit, as ſeen from the Earth's center; or the difference between her true place on the Sun, as ſeen from the Earth's center, and her apparent place thereon as ſeen from *London*. Then, meaſure the line *ld* upon the ſame ſcale, and you will find it to be 16″; and ſo much is the eaſtern parallax of Venus on the Sun, at *London*, when the tranſit ends at the Earth's center. Therefore Venus will be 16″ ſhort of her beginning of egreſs from the Sun, at *London*, when her egreſs begins at the Earth's center.

62. The eaſtern mouth of the *Ganges* is 89 degrees eaſt of the meridian of *London*. Therefore, when it is 28 minutes paſt II in the morning at *London*, (§ 59) it will be 24 minutes paſt VIII in the morning at the *Ganges*; and when it is 20 minutes paſt VIII at *London*, it will be 26 minutes paſt II in the afternoon at the *Ganges*.

Therefore, when the tranſit begins at the Earth's center, the eaſtern mouth of the *Ganges* will be at *G*, as ſeen from the Sun, and at *g* when the tranſit ends.

Draw *Ge* and *gh*, each perpendicular to the meridian *NCS*, and meaſure them on the ſcale: the former will be 22¼ ſeconds for the eaſtern parallax of Venus from the Sun, as ſeen from the *Ganges*, when the tranſit begins at the Earth's center; and the latter will be 16½ ſeconds for the weſtern parallax of Venus from the Sun, when it ends.

63. *Bencoolen* is 102 degrees eaſt of the meridian of *London*, and therefore, when the time is 28 minutes paſt II in the morning at *London*, it is 16 minutes paſt IX in the morning at *Bencoolen*; and when it is 20 minutes paſt VIII in the morning at *London*, it is 8 minutes paſt III in the afternoon at *Bencoolen*. Therefore, as ſeen from the Sun, *Bencoolen* will be at *B* when the tranſit begins at the Earth's center; and at *b* when it ends.

Draw *Bi* and *bk*, each perpendicular to the meridian *NCS*, and meaſure their lengths on the ſcale *AB*; the former will be found 19½ ſeconds for the eaſtern parallax of Venus from the Sun, as ſeen from *Bencoolen*, when the tranſit begins at the Earth's center; and the latter will be 22½ ſeconds for the weſtern parallax, when it ends.

64. Only the end of the tranſit will be viſible at *St. Helena*, which place being 6 degrees weſt of the meridian of *London*, when it is 20 minutes paſt VIII in the morning at *London*, it will be only 56 minutes paſt VII in the morning at *St. Helena*. Therefore, as ſeen from the Sun, *St. Helena* will be at *H* when the tranſit ends as ſeen from the Earth's center.

Draw *Hn* perpendicular to the meridian *NCS*, and meaſure its length in the ſcale *AB*, which you will find to be 26 ſeconds, for the eaſtern parallax of Venus on the Sun, as ſeen from *St. Helena*, when the tranſit ends as ſeen from the Earth's center.

65. To all theſe places, the parallax of Venus is eaſtward in the forenoon, and weſtward in the afternoon.—Therefore, Venus will be later in coming to any given point of the Sun's diſc in the forenoon, as ſeen from theſe places, than as ſeen from the Earth's center; and ſhe

will come ſooner to any given point of the Sun's diſc in the afternoon, as ſeen from them, than as ſeen from the Earth's center: for a parallax to the eaſt keeps her back, and a parallax to the weſt carries her forward. If this be not well attended to, we may make wrong concluſions in ſome particulars.—To avoid which, let it be remembered, that as the beginning of the tranſit is inviſible both at *London* and *St. Helena*, and the end of it is in the forenoon at both theſe places, Venus will have an eaſtern parallax, as ſeen from both of them, when the tranſit ends at the Earth's center; and therefore, its ending at them will be later than at the Earth's center. At the *Ganges* and *Bencoolen*, the parallax is eaſtward when the tranſit begins, becauſe the time is in their forenoon; and weſtward when it ends, becauſe the time is in their afternoon. Conſequently, the tranſit will begin later, and end ſooner, at theſe two places, than at the Earth's center.

66. We now collect the above times and parallaxes, in order to have them at hand in computing their influences on the duration of the tranſit. E ſignifies eaſt, W weſt, M morning, A afternoon.

		Beginning.	Parallax.	Ending.	Parallax.
		H. Min.	"	H. Min.	"
At	The Earth's center	II 28 M.	0	VIII 20 M.	0
	London	Inviſible	—	VIII 20 M.	16 E.
	Ganges	VIII 24 M.	22¾ E.	II 16 A.	16½ W.
	Bencoolen	IX 16 M.	19½ E.	III 8 A.	22½ W.
	St. Helena	Inviſible	—	VII 56 M.	26 E.

67. If the viſible line of the tranſit, or path of Venus over the Sun's diſc, were the ſame as ſeen from theſe four places on the Earth's ſurface, its durations at them could be affected only by the quantities of theſe parallaxes from their meridians.—We ſhall at preſent ſuppoſe it to be the ſame; and after having computed the quantities of contraction and protraction in time, ariſing from the ſaid parallaxes, we ſhall ſhew in the next article what allowances muſt be made on account of the different lengths of the tranſit-line, ariſing from the parallaxes in latitude of Venus on the Sun's diſc; which is northward to thoſe places that are on the ſouth ſide of the plane of the ecliptic on the Earth's diſc, and ſouthward to thoſe places which are on the north ſide thereof.

68. It appears by the theory of Venus, that her true motion on the Sun, as ſeen from the quieſcent center of the Earth, will be 4 minutes (or 240″) of a degree *per* hour; therefore, in a minute of time ſhe

will run over 4 ſeconds on the Sun's diſc, and, conſequently, over 1 ſecond in 15 ſeconds of time.

69. At *London*, the parallax is 16″ eaſtward when the tranſit ends at the Earth's center. Say therefore, as 240″ is to one hour or 60 minutes of time, ſo is 16″ to 4 minutes of time; and ſo much later would the tranſit end at *London* than at the Earth's center, if it was only affected by this ſingle parallax: and hence, the ending at *London* would be at VIII hours 24 minutes in the morning. *See* § 65 *and* 66, *in this and the three following caſes.*

70. At the eaſtern mouth of the *Ganges*, the parallax is 22¾″ eaſtward, when the tranſit begins at the Earth's center. Therefore, as 240″ is to 60 minutes, ſo is 22¾″ to 5 minutes 41 ſeconds of time; and ſo much later would the tranſit begin at the *Ganges* than at the Earth's center, if nothing beſides this parallax affected it: and conſequently, the beginning at the *Ganges* would be at VIII hours 29 minutes 41 ſeconds in the morning.

At the ſame place, the parallax is 16½″ weſtward when the tranſit ends at the Earth's center. Therefore, as 240″ is to 60 minutes, ſo is 16½″ to 4 minutes 7 ſeconds of time; and ſo much ſooner would the tranſit end in the *Ganges* than at the Earth's center. Hence, the ending at the *Ganges* would be at II hours 11 minutes 53 ſeconds in the afternoon.

Thus, the whole contraction of duration of the tranſit at the *Ganges*, by the eaſtern and weſtern parallaxes of Venus from the Sun, is 9 minutes 48 ſeconds of time (*viz.* 5 minutes 41 ſeconds at the beginning, and 4 minutes 7 ſeconds at the end) and ſo much ſhorter will the duration be at the *Ganges* than as ſeen from the Earth's center.

71. At *Bencoolen*, the parallax is 19½″ eaſt when the tranſit begins at the Earth's center. Therefore, as 240″ is to 60 minutes of time, ſo is 19½″ to 4 minutes 52 ſeconds of time; and ſo much later would the tranſit begin at *Bencoolen* than at the Earth's center. So that, by this parallax, the beginning would be at IX hours 20 minutes 52 ſeconds in the morning.

At the ſame place, the parallax is 22½″ weſtward when the tranſit ends at the Earth's center. Therefore, as 240″ is to 60 minutes, ſo is 22½″ to 5 minutes 37 ſeconds of time; and ſo much ſooner would the tranſit end at *Bencoolen* than at the Earth's center: hence, the ending, by this parallax, would be at III hours 2 minutes 23 ſeconds in the afternoon.

So that the whole contraction of duration at *Bencoolen*, by the eaſtern parallax at the beginning of the tranſit, and the weſtern parallax at the end, would be 10 minutes 29 ſeconds; which is 41 ſeconds more than at the mouth of the *Ganges*.

72. At *St. Helena*, the parallax is 26″ eaſtward when the tranſit ends at the Earth's center. Therefore, as 240″ is to 60 minutes of time, ſo is 26″ to 6½ minutes of time; and ſo much later would the tranſit end at *St. Helena*, by means of this parallax, than at the Earth's center. Hence, the ending at *St. Helena* would be at VIII hours 2 minutes 30 ſeconds in the morning.

73. Having ſhewn the influences of the eaſtern and weſtern parallaxes of Venus on the duration of the tranſit, we come next to ſhew what muſt ariſe from her northern and ſouthern parallaxes, which will be rendered more intelligible by a projection of the viſible lines of the tranſit on the Sun's diſc, than it could poſſibly be by calculations.

ARTICLE VIII.

Shewing how to project the tranſit of Venus on the Sun's diſc, as ſeen from different places of the Earth; and to make the allowances mentioned in § 67.

74. The proper elements for this conſtruction are as follow:

I. The true time of conjunction of the Sun and Venus, as ſeen from the Earth's center, and reckoned according to the equal time at *London*, on the 6th of June 1761, is at V hours 46 minutes in the morning, according to Dr. HALLEY's tables.

II. The geocentric, or viſible latitude of Venus at that time 9′ 43″ ſouth.

III. The angle of the tranſit-line with the ecliptic, as ſeen from the Earth's center, 8° 31′; the angular point or deſcending node being eaſtward from the Sun, as ſeen from the Earth; the Sun in 15° 35′ 55″ of Gemini, and the aſcending node in Gemini 14° 29′ 37″.

IV. The Sun's ſemidiameter on the day of the tranſit, 15′ 50″.

V. The ſemidiameter of Venus (from the Doctor's Diſſertation) 37½″.

VI. The viſible difference of the ſemidiameters of the Sun and Venus 15′ 12½″.

VII. Their ſum, 16′ 27½″.

VIII. The inclination of the axis of Venus's viſible path to the axis of the ecliptic, 8° 31′; the ſouthern half of the axis of her path lying to the left hand of the ſouthern half of the axis of the ecliptic, as ſeen from the Earth.

IX. The true motion of Venus in the line of her tranſit, as ſeen from the Earth's center, 4 minutes of a degree *per* hour.

X. The inclination of the Earth's axis to the axis of the ecliptic, 6°; the ſouthern half of the Earth's axis lying to the left hand of the ſouthern half of the axis of the ecliptic, as ſeen from the Sun.

XI. The inclination of the axis of Venus's path to the Earth's axis, 14° 31′.

75. Theſe elements being collected, make a ſcale of any convenient length, as that of Fig. 1. in Plate III. and divide it into 17 equal parts, each whereof ſhall be taken for a minute of a degree: then, divide the minute next to the left hand into 60 equal parts for ſeconds, by diagonal lines, as in the figure.

76. Draw *ACG* (Fig. 4.) for a ſmall portion of the ecliptic, and perpendicular thereto draw *CvE* for the axis of the ecliptic.

77. Take the Sun's ſemidiameter 15′ 50″ in your compaſſes from the ſcale, and with that extent as a radius, ſetting one foot in *C* as a center, deſcribe the ſemicircle *AEG* for the ſouthern half of the Sun's diſc, becauſe the tranſit is on that half of the Sun.

78. Take the geocentric latitude of Venus at the conjunction, 9′ 43″, in your compaſſes from the ſcale, and ſet that extent from *C* to *v* on the axis of the ecliptic; becauſe the viſible latitude of Venus is ſo far ſouth of the Sun's center, at the moment of her conjunction with the Sun.

79. Draw the right line *CtD*, making an angle of 8° 31′ with the axis of the ecliptic, to the left hand, and this line ſhall be the axis of Venus's viſible path over the Sun's diſc.

80. Through the point of conjunction *v*, in the axis of the ecliptic, draw the right line *qtr* for the viſible path of Venus, or line of her tranſit over the Sun, at right angles to the axis of her orbit *CtD*; which will divide the line of her tranſit into the two equal parts *tq* and *tr*.

81. Take Venus's horary motion on the Sun, 4′, from the ſcale with your compaſſes; and with that extent make marks on the tranſit line *qtr*. The equal ſpaces from mark to mark ſhew how much of that line Venus moves through in each hour, as ſeen from the Earth's center, whilſt ſhe continues on the diſc of the Sun.

82. Divide each of theſe horary ſpaces, from mark to mark, into 60 equal parts or minutes, and ſet the hours to the proper marks, in ſuch a manner, that the true time of conjunction of the Sun and Venus, which is 46 minutes after V in the morning, may fall into the point *v*, where the tranſit-line cuts the axis of the ecliptic: ſo the points *v* ſhall be the inſtant of conjunction; and *t* the middle of the tranſit, which is at 24 minutes after V in the morning, as ſeen from the Earth's center, and reckoned by the time at *London*.

83. Take the difference between the ſemidiameters of the Sun and Venus, $15' \, 12\frac{1}{2}''$, from the ſcale with your compaſſes; and with that extent, ſetting one foot in the Sun's center *C*, deſcribe the arcs *N* and *T* with the other, cutting the tranſit-line in the points *k* and *l*, which are thoſe on the Sun's diſc that are hid by the center of Venus, at the moments of her two internal contacts with the Sun's limb at *M* and *N*: the former of theſe is the moment of the total ingreſs of Venus upon the Sun, *viz.* at II hours 28 minutes in the morning, as ſeen from the Earth's center; and the latter is the moment when her egreſs from the Sun begins, as ſeen from the Earth's center, *viz.* at VIII hours 20 minutes in the morning, according to the time at *London*. The interval between theſe times is 5 hours 52 minutes.

84. The central ingreſs of Venus on the Sun is the moment when her center is directly even with the Sun's eaſtern limb, which is at *u*, at 15 minutes after II in the morning; and her central egreſs from the Sun is the moment when her center is directly even with the Sun's weſtern limb, which is at *w*, 33 minutes after VIII in the morning, as ſeen from the Earth's center, and reckoned by the time at *London*. The interval between theſe times is 6 hours 18 minutes.

85. Take the ſum of the ſemidiameters of the Sun and Venus, $16' \, 27\frac{1}{2}''$, in your compaſſes from the ſcale; and with that extent, ſetting one foot in the Sun's center *C*, deſcribe the arcs *Q* and *R* with the other, cutting the line of the tranſit in the points *q* and *r*. Theſe are the points in open ſpace (clear of the Sun) where the center of Venus is, at the moments of her two external contacts with the Sun's limb, at *S* and *W*; or the moments of the beginning and end of the tranſit as ſeen from the Earth's center; the former of which is at 3 minutes after II in the morning at *London*, and the latter at 45 minutes after VIII. The interval between theſe times is 6 hours 42 minutes.

86. Take the ſemidiameter of Venus, $37\frac{1}{2}''$, in your compaſſes from the ſcale; and with that extent, on the points *q*, *k*, *l*, *r*, and *t*, as centers, deſcribe the circles *HS*, *MI*, *PN*, *WY*, and *OF*, for the diſc

of Venus, at her firſt contact *S*, her total ingreſs at *M*, her beginning of egreſs at *N*, her laſt contact at *W*, and her place on the Sun at *OF* at the middle of the tranſit.

87. Draw the right line *Cf*, making an angle of 6° with the axis of the ecliptic *CvE* toward the right hand in this projection, which will be to the left as ſeen from the Sun. So *Cf* ſhall be the ſouthern half of the Earth's axis produced to the whole length of the Sun's ſemidiameter, and making an angle of 14° 31′ with the axis of Venus's orbit *CtD*. The uſe of the Earth's axis in this projection is for thoſe who may chooſe to draw the Earth's diſc round *C* as a center, with the radius 31″ (which is the difference between the horizontal parallaxes of the Sun and Venus) and lay down the paths of places therein, in order to find the times of the beginning and ending of the tranſit at theſe places, according to the Doctor's method.

88. We come now to the method of finding Venus's parallax of latitude on the Sun, as ſeen from different places on the Earth's ſurface; and to ſhew its influence on the duration of the tranſit as ſeen from theſe places, according as their ſituations on the Earth's diſc are north or ſouth of the great circle in which it is cut by the plane of the ecliptic, both when the tranſit begins and ends as ſeen from the Earth's center.

89. It appears, by what has been ſaid in § 61. concerning Fig. 2. of Plate III, that, as ſeen from the Sun, *London* will be at *L* on the Earth's diſc, at the middle of the tranſit, and at *l* when it ends: at both which, and the intermediate times, *London* is on the north ſide of the plane of the ecliptic *ACB*; but nearer to that plane at *l*, than at *L*. Therefore, the line of the tranſit, as ſeen from *London*, will be to the ſouth of the true line of the tranſit, as ſeen from the Earth's center, but nearer thereto at the end than at the middle. For the north latitude of a place from the plane of the ecliptic will depreſs the viſible latitude of Venus on the Sun by a ſouthern parallax; and the ſouth latitude of a place from the plane of the ecliptic will elevate the viſible latitude of Venus on the Sun by a northern parallax. *See* § 12 *and* 13.

90. The mouth of the *Ganges* is at *G*, a little on the north ſide of the ecliptic, at the beginning of the tranſit; very near it at the middle; and at *g* directly in its plane at the end. Therefore, the viſible tranſit at the *Ganges* will be a little ſouth of the central tranſit from the beginning to the middle, and coincident with it from the middle to the end.

91. *Bencoolen* at *B* is ſouth of the ecliptic at the beginning of the tranſit, and alſo at *I* when it ends; but ſtill more ſo at the middle.

Therefore, the whole viſible line of the tranſit at *Bencoolen* will be north of the central tranſit-line: and, ſince the motion of *Bencoolen* is in a curve as ſeen from the Sun, Venus will move over the Sun's diſc in a curve bending the contrary way, as ſeen from *Bencoolen*; a little farther north of the central tranſit at the middle than at the beginning and end, but ſomewhat nearer at the beginning than at the end.

92. *St. Helena* (where only a ſmall part of the tranſit is viſible) will be at *H* when the tranſit ends; and then, its diſtance from the plane of the ecliptic will be ſo nearly the ſame as that of *Bencoolen*, that the difference is not worth any notice.

93. Draw *Lt* and *lu*, each perpendicular to the plane of the ecliptic *ACB*, Fig. 2. of Plate III, and meaſure their lengths on the ſcale *AB* (Fig. 3.) the former is 25″ for the ſouthern parallax of Venus on the Sun's diſc, as ſeen from *London* at the middle of the tranſit; and the latter is almoſt 20″ for the ſouthern parallax of Venus on the Sun when the tranſit ends.

94. Draw *Gf* perpendicular to the plane of the ecliptic, and meaſure its length on the ſcale *AB*, which will be 6½″ for the ſouthern parallax of Venus on the Sun when the tranſit begins, as ſeen from the *Ganges*. There is no parallax in latitude to that place when the tranſit ends, becauſe it is then in the plane of the ecliptic at *g*.

95. Draw *Bv* and *bm* perpendicular to the plane of the ecliptic, and meaſure them on the ſcale *AB*: the former is 8½″ for the northern parallax of Venus on the Sun when the tranſit begins at *Bencoolen*; and the latter is 12″ for the northern parallax at the end.—At the middle, the northern parallax is almoſt 14″.

96. Draw *Ho* perpendicular to the plane of the ecliptic, and meaſure it on the ſcale, which will be 11″ for the northern parallax at *St. Helena*, when the tranſit ends.

97. If theſe parallaxes (§ 93, 94, 95, 96) be taken in ſeconds from the ſcale (Fig. 1.) and ſet off, parallel-wiſe to the axis of the ecliptic (Fig. 4.) from the central tranſit line *ktl*, below or above it, according as the places themſelves are above or below the plane of the ecliptic in Fig. 2. and lines, as *oLp*, *ctl*, and *aBb* be drawn through theſe points of diſtance, they will be the viſible lines of the tranſit as ſeen from *London*, the *Ganges*, *Bencoolen*, and *St. Helena*: to the two laſt it is ſo nearly the ſame, that the difference may be neglected in ſo ſmall a figure.

98. Now, as the arc *N* cuts all theſe lines in the points at which the center of Venus will be on the Sun, at the moment of her total ingreſs

within his eaſtern limb, as ſeen from the above places; and the arc *T* cuts them all in the points where the center of Venus will be ſeen on the Sun at the moment when her egreſs begins at his weſtern limb; it is eaſy to find how much ſhorter or longer the viſible line of the tranſit is, from the total ingreſs to the beginning of egreſs, at thoſe places, than the true tranſit-line *ktl* as ſeen from the Earth's center. For,

I. If the laſt half *Lp* of the tranſit for *London* be taken in the compaſſes, and meaſured from *t* towards *l* on the central tranſit-line, it will be found to be 5 minutes ſhorter. Therefore, the parallax in latitude will ſhorten the tranſit at its ending at *London* by 5 minutes of time.

In § 69, it was found, that by the eaſtern parallax of Venus from the Sun, the tranſit would end at *London* at 24 minutes after VIII in the morning. From which time, ſubtract the affection by the parallax of latitude, *viz.* 5 minutes, and it gives the time of ending of the tranſit at *London*, namely, at VIII hours 19 minutes in the morning.

II. The firſt half *tc* of the tranſit at the *Ganges*, meaſured as above, is a minute ſhorter than the firſt half *tk* of the central tranſit line; but the laſt half *tl* is of the ſame length with that of the central tranſit, becauſe it is coincident therewith. Therefore, the viſible tranſit at the *Ganges* will be contracted only one minute of time at the beginning, and nothing at all at the end, by the parallax of latitude.

Hence, if to the time mentioned in § 70 we only add a minute at the beginning, we ſhall have the time when the tranſit begins and when it ends at the *Ganges*; ſo that the beginning will be at VIII hours 30 minutes 41 ſeconds in the morning, and the ending will be at II hours 11 minutes 53 ſeconds in the afternoon.

III. The firſt half *aB* of the viſible tranſit-line at *Bencoolen* will be found 2 minutes longer, and the laſt half *Bb* 2$\frac{1}{2}$ minutes longer, than that of the central tranſit-line, by means of the parallax of latitude.

Therefore, if we ſubtract 2 minutes from the time the tranſit begins at *Bencoolen* by the eaſtern parallax of Venus from the Sun, and add 2 minutes 30 ſeconds to the time of its ending by that parallax, as mentioned in § 71, we ſhall have the viſible time of the beginning and ending of the tranſit at *Bencoolen*, viz. the beginning at IX hours 18 minutes 52 ſeconds in the morning, and the ending at III hours 4 minutes 53 ſeconds in the afternoon.

IV. The end *b* of the tranſit-line at *St. Helena* falls nearly in with its end at *Bencoolen*: and therefore, we have only 2 minutes 30 ſeconds for the parallax of latitude to add to the time of ending of the tranſit mentioned in § 72: and hence, the viſible time of ending at *St. Helena* will be at VIII hours 5 minutes in the morning.

99. Let theſe matters be collected into a little table, ſhewing at what times the tranſit begins and ends at the places themſelves; and what the ſame inſtants are at *London*, by reducing the times according to the difference of meridians. *See* § 62, 63, 64.—The beginning is inviſible at *London*, but the ending is at VIII hours 18 minutes in the morning.

The times at theſe Places.	Beginning. H. M. S.	Ending. H. M. S.	At *London*. Beginning. H. M. S.	Ending. H. M. S.
The *Ganges*	VIII 30 41 M.	II 11 53	II 34 41 M.	VIII 15 53 M.
Bencoolen	IX 18 52 M.	III 4 53	II 30 52 M.	VIII 16 53 M.
St. Helena	Inviſible M.	VIII 5 0	Inviſible M.	VIII 29 0 M.

100. And now I preſume, that moſt of my readers will think theſe are the true times of the beginning and ending of the tranſit; which indeed they would be, if it fell on the 21ſt of June, all other circumſtances being as above related. For then, the eaſtern and weſtern parallaxes of Venus from the meridian *NCS* (Fig. 2.) would be the ſame with her parallaxes in longitude, becauſe the plane of the meridian would coincide with the axis of the ecliptic *CED*. But as that is by no means the caſe on the 6th of June (the day of this tranſit) when the Earth's axis is inclined to the axis of the ecliptic in the projection; in order to have the true times, we muſt take the neareſt diſtances of the above places from the axis of the ecliptic (when the tranſit begins and when it ends at the Earth's center) and meaſure them, as above directed, on the ſcale *AB* (Fig. 3.) for the true parallaxes of Venus in longitude from the Sun; and reduce them into time, as ſhewn in § 69, 70, 71, 72; and apply them as there directed, together with the parallaxes in latitude mentioned § 98.

I. Thus, the neareſt diſtance of *London (l)* from the axis of the ecliptic *(CED)* when the tranſit ends at the Earth's center, being meaſured on the ſcale, is 14″ for the parallax in longitude; which being reduced to time makes 3 minutes 30 ſeconds, to be added to the ending of the tranſit at *London* mentioned in § 66, *viz.* VIII. hours 20 minutes in the morning, and it gives VIII hours 23 minutes 30 ſeconds; from which ſubtract 5 minutes for the parallax of latitude (§ 98, N° I.) and there will remain VIII hours 18 minutes 30 ſeconds in the morning, for the true time of ending of the tranſit at *London*.

II. The difference between the parallaxes in longitude and parallaxes from the meridian, at the *Ganges*, is ſo ſmall, that it is hardly worth notice; eſpecially as no obſervations are to be made at that place.

III. The neareſt diſtance of *Bencoolen (B)* from the axis of the ecliptic being taken in the compaſſes, and meaſured upon the ſcale, is 21″ for the parallax of the longitude at the beginning of the tranſit, anſwering to 5 minutes 15 ſeconds in time; which being added to the time of beginning at *Bencoolen* mentioned in § 66, *viz.* IX hours 16 minutes in the morning, gives IX hours 21 minutes 15 ſeconds; from which ſubtract 2 minutes for the parallax of latitude (§ 98, N° III.) and there remains IX hours 19 minutes 15 ſeconds, for the true time of the beginning of the tranſit at *Bencoolen.*

The ſame place, being at *b* when the tranſit ends at the Earth's center, is 22″ from the axis of the ecliptic; which being reduced to time is 5 minutes 30 ſeconds for the parallax of longitude; this ſubtracted from the ending of the central tranſit at *Bencoolen*, (§ 66) *viz.* III hours 8 minutes in the afternoon, leaves III hours 2 minutes 30 ſeconds; to which add 2 minutes 30 ſeconds for the parallax of latitude (§ 98, N° III.) and it gives III hours 5 minutes 0 ſeconds for the true time of ending of the tranſit at *Bencoolen.*

IV. The leaſt diſtance of *St. Helena (H)* from the axis of the ecliptic, when the tranſit ends at the Earth's center, is 27″; which being reduced to time, is 6 minutes 45 ſeconds for the parallax of longitude, to be added to the ending of the central tranſit at *St. Helena*, viz. VII hours 56 minutes (§ 66); and the parallax of latitude 2 minutes 30 ſeconds (§ 98, N° IV.) is alſo to be added; which, in all, gives VIII hours 5 minutes 15 ſeconds, for the true time of ending of the tranſit in the morning, at *St. Helena.*

And theſe times may be reduced to the times at *London*, by ſubtracting 6 hours 48 minutes from the times of beginning and ending at *Bencoolen*, and adding 24 minutes to the time of ending at *St. Helena*; theſe being the temporary difference of their meridians from the meridian of *London.*

101. All this is on ſuppoſition that the Sun's horizontal parallax is $12\frac{1}{2}''$, and that the tables do give us the exact time of conjunction of the Sun and Venus, and alſo her true latitude. I have particular reaſons for believing that the latitude is accurate, but that the abovementioned times will be found to anticipate the true obſerved times by about 5 minutes, and that the diameter of Venus on the Sun will not be found to ſubtend an angle of more than 1′ 2″.

102. It ſeems that *Bencoolen* is the only place where the Engliſh are to make obſervations on the whole duration of the tranſit; which, according to the above computations, will be 5 hours 45 minutes 45 ſeconds. From this we are to ſubtract 4 minutes 30 ſeconds for the affection of the tranſit by the parallax of latitude, and then there will remain 5 hours 41 minutes 15 ſeconds; which being ſubtracted from the true duration of the tranſit as ſeen from the Earth's center, *viz.* 5 hours 52 minutes, there will remain 10 minutes 45 ſeconds for the whole contraction of duration, ariſing from the parallaxes in longitude of Venus from the Sun at the beginning and ending of the tranſit, as ſeen from the Earth's center. And this 10 minutes 45 ſeconds is all that we have to go to work with, for finding the true quantity of the Sun's horizontal parallax; inſtead of 17 minutes, which DR. HALLEY expected from the contrary motions of the *Ganges* and *Port-Nelſon*, together with a conſiderably longer tranſit ariſing from the ſmaller quantity of Venus's latitude.—So that now, we ſhall have only an advantage in proportion to that which the Doctor expected, as 10 minutes 45 ſeconds (or 645 ſeconds) is to 17 minutes (or 1020 ſeconds.) Therefore, as 1020 is to 500, ſo is 645 to 316 nearly. Hence, inſtead of being able to find the Sun's parallax to within its 500dth part, we muſt reſt ſatisfied at preſent with being able to find it to within its 316th part: which indeed is ſurprizingly near, in compariſon of the former methods, which could never come nearer than a 10th or 12th part at moſt.

103. If the whole contraction of duration at *Bencoolen*, ariſing both from the parallaxes in longitude and latitude of Venus, is found by obſervation to be 6 minutes 15 ſeconds (or 375 ſeconds) it will prove the Sun's horizontal parallax to be $12\frac{1}{2}''$, provided the geocentric latitude of Venus be 9′ 43″ ſouth, at the inſtant of her conjunction with the Sun, which may be found by obſervation; for, ſuch a parallax of the Sun, and ſuch a latitude of Venus, muſt neceſſarily produce ſuch a contraction in the duration of the tranſit: and then, by § 38, the horizontal parallax of Venus's muſt be $43\frac{9}{10}''$.—If the contraction is found to be greater or leſs, the parallaxes of the Sun and Venus muſt be greater or leſs accordingly. Thus, ſuppoſing the whole contraction at *Bencoolen* ſhould only amount to 5 minutes 15 ſeconds (or 315 ſeconds) then, the Suns horizontal parallax can be only $10\frac{1}{2}''$, and the horizontal parallax of Venus only $36\frac{9}{10}''$. For, as 375 ſeconds of time is to $12\frac{1}{2}''$ of parallax, ſo is 315 ſeconds of time to $10\frac{1}{2}''$ of parallax.

And having thus gained the quantity of the Sun's horizontal parallax from the contraction of the tranſit, his diſtance from the

Earth may be found by the analogies in § 15, and the diſtances of all the other planets from the Sun by § 1 and 2.

104. Having reaſon to believe, that the tables give the time of conjunction of the Sun and Venus 5 minutes ſooner than it will be, that Dr. HALLEY has ſtated the ſemidiameter of Venus at leaſt 6″ too much in the latter part of his Diſſertation, and that the Sun's horizontal parallax is much about $10\frac{1}{2}''$; I ſhall now take the liberty to give my own opinion concerning the times of the beginning and ending of the tranſit, according to a large projection thereof on a ſolar diſc 30 inches in diameter, conſtructed from the following elements. 1. That the true conjunction of the Sun and Venus will be at 51 minutes after V in the morning at *London*. 2. That the true geocentric latitude of Venus at that time will be 9′ 43″ ſouth. 3. That the Sun's ſemidiameter is then 15′ 50″. 4. That the ſemidiameter of Venus is 31″. 5. That the horizontal parallax of Venus from the Sun is $26\frac{4}{10}''$. 6. That the Earth's axis will then be inclined to the axis of Venus's orbit $14\frac{1}{2}$ degrees. And, 7. That the true motion of Venus on the Sun will be 4′ of a degree *per* hour, as ſeen from the Earth's center.

And hence, the abſolute times of Venus's total ingreſs on the Sun, and of her beginning of egreſs from him, reduced to the meridian of *London*, and the different durations of the tranſit from ingreſs to egreſs, will be as follows.

		Beginning.			End.			Duration.		
		H.	Min.	Sec.	H.	Min.	Sec.	H.	Min.	Sec.
At	The Earth's center	II	31	0 M.	VIII	27	0 M.	5	56	0
	London - - -	II	37	32 M.	VIII	26	8 M.	5	48	26
	Ganges - - -	II	35	9 M.	VIII	24	15 M.	5	49	6
	Bencoolen - - -	II	33	29 M.	VIII	24	14 M.	5	50	45
	St. Helena - -	II	36	38 M.	VIII	34	34 M.	5	57	56

Which times, being reduced to the meridians of the three laſt places, are at the places themſelves as follows. M ſignifies morning, A afternoon.

		Beginning.			End.			Duration.		
		H.	Min.	Sec.	H.	Min.	Sec.	H.	Min.	Sec.
At	The *Ganges*	VIII	31	9 M.	II	20	15 A.	5	49	6
	Bencoolen -	IX	21	29 M.	III	12	14 A.	5	50	45
	St. Helena -	II	12	38 M.	VIII	10	34 M.	5	57	56

So that the whole effect of the parallaxes, both in longitude and latitude, will be, to contract the duration of the tranſit 7 minutes 34 ſeconds at *London*, 6 minutes 54 ſeconds at the *Ganges*, 5 minutes 15 ſeconds at *Bencoolen*, and to protract the duration 1 minute 56 ſeconds at *St. Helena*.

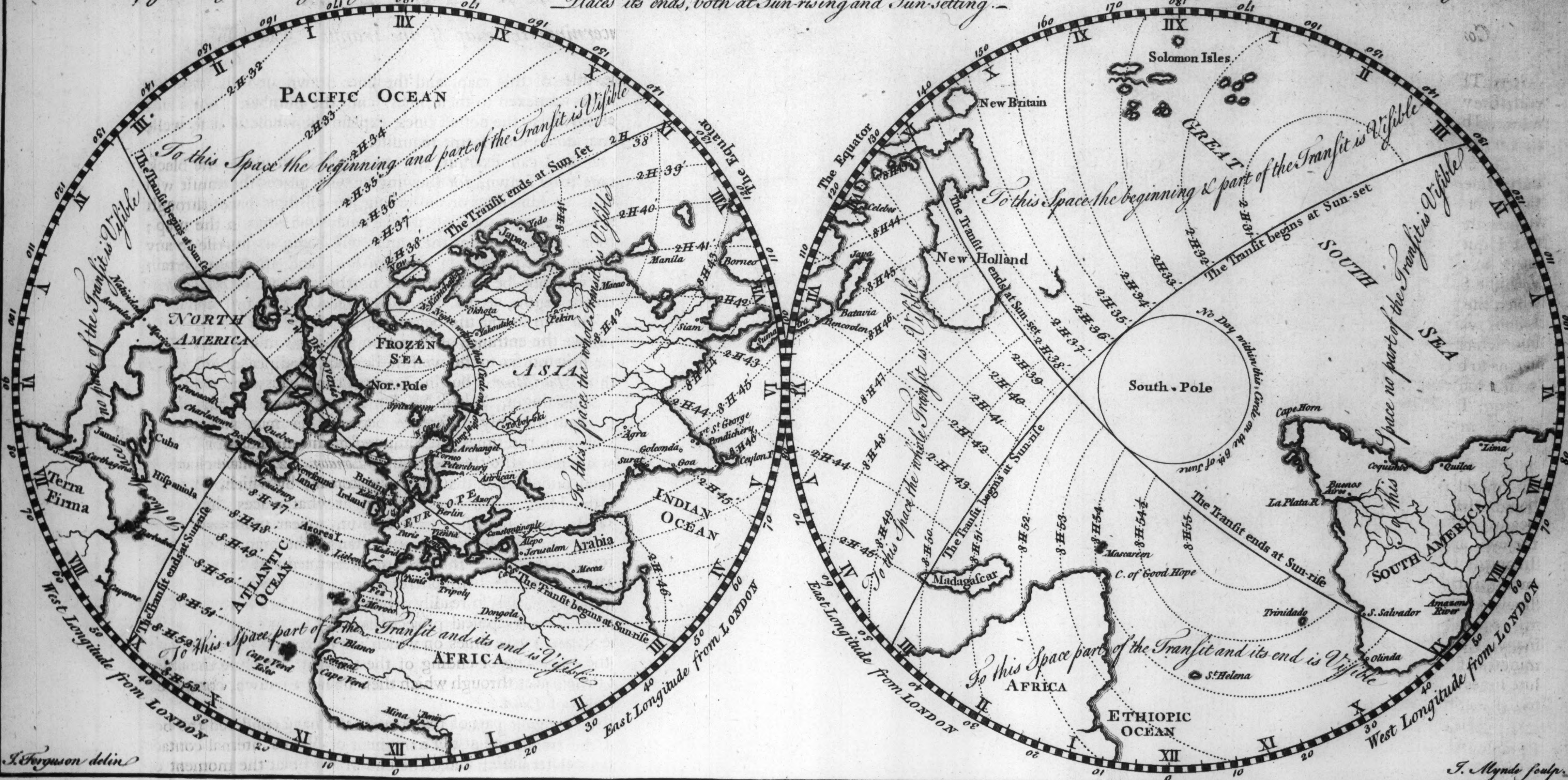
From the firſt contact to the total ingreſs, and from the beginning of egreſs to the laſt contact, will be about 20 minutes.

105. *St. Helena* is a very proper place for obſerving this tranſit at, even though the end can only be ſeen there: for at that time, the parallax of Venus from the Sun will be almoſt equal to the whole quantity of her horizontal parallax from him. And by comparing the obſerved times of ending of the tranſit at *London*, *Bencoolen*, and *St. Helena*, the differences thereof will be very good helps toward the ſolution of this moſt deſirable problem.

106. In the year 1769, there will be another tranſit of Venus over the Sun, much more convenient for inveſtigating the Sun's parallax; but no part of it will be viſible in *Britain*. For the time of conjunction of the Sun and Venus will be on the 3d of June, at a quarter paſt X in the evening at *London*, at which time it will be midnight at *Torne* in *Lapland:* but as the Sun will then be only 2 degrees below the horizon of that place, the whole of the tranſit will be viſible, excepting only a little about the middle, which is of no ſignification. At that time, the Sun will be vertical in the *Great South-Sea*, in north latitude 22⅓ degrees, and eaſt longitude from the meridian of *London* 170 degrees. At that time, it will be about half an hour paſt ten in the morning at the *Solomon Iſles*. If proper obſervers are ſent to *Torne*, and to ſome of theſe iſlands, they will have the like advantage as was mentioned by Dr. HALLEY with regard to the *Ganges* and *Port-Nelſon*. For the *Solomon Iſles* will have a contrary motion to that of Venus, which will accelerate her motion on the Sun as ſeen from them; and *Torne* will have a motion the ſame way as that of Venus, which will retard her motion on the Sun as ſeen from it. And conſequently, there will be almoſt the greateſt poſſible difference in the duration of the tranſit at theſe two places.—It were much to be wiſhed, that our Aſtronomers would neither ſpare coſt nor labour in duly obſerving this tranſit, as there will not be ſuch another opportunity in 100 years afterward.

PL. IIII.

A MAP of the EARTH, *Upon which are marked the Hours and Minutes of true Times of the Entrance and Exit of* VENUS, *in its Paſsage over the* SUN's DISC, June 6th 1761. *The Times are computed to the Meridian of* LONDON, *and that which is expreſsed upon each dotted Line shews when the Tranſit begins or ends at all those Places which the same Line paſses through. The beginning is between* II *and* III *in the Morning and the end between* VIII *and* IX. *The four black curve Lines shew at what Places the Tranſit begins, and at what Places its ends, both at Sun-rising and Sun-setting.*



ARTICLE IX.

Concerning the map of the tranſit. Plate IV.

107. The title of this map, and the lines drawn upon it, together with the words annexed to theſe lines, and the numbers [hours and minutes] belonging to the dotted lines, explain the whole of it ſo well, that no farther deſcription ſeems requiſite.

108. So far as I can examine the map by a good globe, the black curve lines are truly drawn, for ſhewing at what places the tranſit will begin, or end, at ſun-riſing or ſun-ſetting, to all thoſe places through which they are drawn, according to the calculated times in the map; but I queſtion much whether the tranſit will begin at ſun-riſe to any place in *Africa*, that is weſt of the *Red-Sea*; and am pretty certain that the Sun will not be riſen to the northmoſt part of *Madagaſcar* when the tranſit begins, as M. De L'Isle reckons the firſt contact of Venus with the Sun to be the beginning of the tranſit. So that the line which ſhews the entrance of Venus in the Sun's diſc at ſun-riſing, ſeems to be a little too far to the weſt in the map, at all places which are to the ſouth of *Aſia Minor*: but in *Europe*, I think it is very well.

109. In delineating it, I had M. De L'Isle's map of the tranſit before me. And the only difference between his map and this is, 1. That in his map, the times are computed to the meridian of *Paris*; in this, they are reduced to the meridian of *London*. 2. I have changed his meridional projection into that of the equatoreal; by which, I apprehend, that the black curve lines, ſhewing at what places the tranſit begins, or ends, with the riſing or ſetting-ſun, appear more natural to the eye, and are more fully ſeen at once, than in the map from which I copied: for, in that map, the lines are interrupted and broke in the meridian that divides the hemiſpheres; and the places where they ſhould join cannot be ſeen ſo readily by thoſe who are not well ſkilled in the nature of ſtereographical projections.—The like may be ſaid of many of the dotted curve lines on which are expreſſed the hours and minutes of the beginning or ending of the tranſit, which are the abſolute times at theſe places through which theſe lines are drawn, computed to the meridian of *London*.

110. In the foregoing part of this treatiſe, I have conſidered the beginning of the tranſit to be at the moment of Venus's internal contact with the Sun's eaſtern limb; and the end of it to be at the moment of

her internal contact with the Sun's weſtern limb; for the reaſon given in § 60. But M. De L'Isle has reckoned the moment of Venus's firſt contact with the Sun's eaſtern limb to be the beginning of the tranſit, and the moment of her laſt contact with the Sun's weſtern limb to be the end of the tranſit; which indeed, properly ſpeaking, it is.

From the moment of the firſt contact to that of the total egreſs or internal contact with the Sun's eaſtern limb, will be about 20 minutes; and from the moment of beginning of egreſs, or laſt internal contact, to that of the total exit of Venus from the Sun, will be much about 20 minutes alſo.

111. By the above calculation of the times in § 104, with the addition of 20 minutes for the total exit of Venus from the Sun, I find, that at *London* the ſaid exit will be at VIII hours 46 minutes 8 ſeconds; at *Bencoolen*, it will be at VIII hours 44 minutes 14 ſeconds; and at *St. Helena* at VIII hours 54 minutes 34 ſeconds. By comparing theſe times with thoſe in the map, I find my calculation 8 ſeconds later than in the map at *London*; 46 ſeconds earlier than in the map at *Bencoolen*; and only about 2 ſeconds earlier than the map gives the time of *St. Helena*. Which of theſe comes neareſt the truth, muſt be determined by obſervation.

112. M. De L'Isle, in a memoir preſented to the French king with his map, propoſes rather to find the Sun's diſtance by accurate obſervations made at different places, on Venus's going off the Sun's diſc, than by Dr. Halley's method of finding the ſame by obſervations on the differences of the whole durations of the tranſit at different places; and points out the *Cape of Good Hope* and *Tobolſk* in *Siberia*, where the difference cauſed by parallax will be 11½ minutes: but he ſays, that if the obſervers could go ſtill father to the eaſt, as to *Yakoutſk* in *Siberia*, the advantage would be ſtill greater, becauſe the difference of time between Venus's exit from the Sun at *Yakoutſk* and at the *Cape of Good Hope* will be 13½ minutes. He alſo mentions, that there are ſome places in the north of *Europe*, as *Wardhus* and *Archangel*, where the ſame advantage would nearly be gained, as at *Tobolſk*. If the Sun's diſtance is to be found in this manner, it will require that the longitudes of theſe places be firſt determined to the greateſt degree of preciſion.

ARTICLE X.

Containing an extract from a paper in the Connoiſſance des Temps *for the year* 1761. *Titled thus:*

Of the paſſage of Venus on the Sun's diſc, to be obſerved the 6th of June 1761, *and the importance of that phenomenon.*

The conjunction of Venus with the Sun is actually the moſt intereſting, as well as the moſt uncommon appearance, that the heavens preſent to us. For more than a century paſt it has been predicted, as to happen in the year 1761, Aſtronomers have been extolling the uſe that may be made of it, and exhorting all who ſhall have the good luck to ſee it to profit as much as poſſible by ſo famous an obſervation.

And indeed, what would not one undertake in the proſpect of this rare phenomenon? whoſe advantages once let ſlip, could not be retrieved by any effort of genius, by the moſt unwearied ſtudy, or even by the encouragement of the greateſt princes. But as it will be neceſſary, if we would make a proper uſe of this tranſit, that it be obſerved in *America* * and in the *Eaſt-Indies*, we found ourſelves obliged to give the following account of it; earneſtly recommending it to the conſideration of all perſons capable of lending their aſſiſtance in the preſent inquiry.

Kepler having, from *Tycho*'s obſervations, compoſed his *Rudolphine* Tables, ſaw very well that Mercury and Venus muſt ſometimes paſs over the diſc of the Sun: and accordingly gave notice of two tranſits of Venus, one in the year 1631, the other in 1639, in a quarto pamphlet publiſhed at *Leipſic* 1629 (*Admonitio ad Aſtronomos rerumque cæleſtium ſtudioſis de miris rariſque anni* 1631 *phenomenis, Veneris putà & Mercurii in Solem incurſu*). *Kepler* died a few days before that which he had fixed for the tranſit of Venus 1631; and *Gaſſendi* who looked for it at *Paris* was diſappointed (*Vid. Mercurius in Sole viſus, & Venus inviſa*). The truth is, a ſmall degree of inaccuracy in the *Rudolphine* Tables gave a tranſit of Venus in 1631, when there was

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[*] I have ſhewn in § 52, that obſervations would profit nothing at *Port Nelſon* in *North America*.

none; and gave no tranſit for 1639, the year in which it actually happened.

Horrox, a young Engliſh Aſtronomer, having amuſed himſelf in calculating Ephemerides from the tables of *Lanſberg* (far leſs correct than thoſe of *Kepler*) concluded from thence, the firſt of any, that Venus would paſs over the Sun in the year 1639; and prepared himſelf to make this obſervation. *Lanſberg*'s tables gave an error of 16′ in latitude, and the *Rudolphine* Tables only an error of 8′. But the error of *Lanſberg* elevated Venus above the Sun's diſc, and that of *Kepler* made her paſs below it. By this means, very bad tables produced an excellent obſervation; they were in eſteem at that time, becauſe they had not been examined; and their author had puffed them with an impoſing aſſurance *. Follows the whole of Horrox's obſervation, extracted from a paper which he drew up in 1640, a little before his death, and which Hevelius publiſhed in 1662, at the end of his *Mercurius in Sole viſus*, pag. 111.

The 24th of November, old ſtyle, Venus entered the Sun's diſc, about 6½ degrees from his nadir, that is, from a verticle line drawn through his center.

At III hours 15 minutes the diſtance of the centers of the Sun and Venus was 15′ 39″, at III hours 35 minutes it was 14′ 40″, and at III hours 45 minutes, the diſtance was 14′ 7″. All this on the ſuppoſition that the Sun's diameter was 32′ 36″; as I find it muſt have been at that time. The diameter of Venus, meaſured ſeveral times, ſeemed to be at moſt 1′ 18″. If this quantity is not quite exact, the error is rather in exceſs; for I am ſure Venus's diameter was not greater than this. And indeed, Mr. *Crabtree*, a friend of Mr. *Horrox*'s who lived about 8 leagues off, to whom he had given notice of this phenomenon, rated it at no more than $\frac{7}{200}$ parts of the Sun's diameter, that is, at 1′ 8″.

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* The *French* writers are particularly noted for their candor and politeneſs toward the learned of other nations. We muſt therefore impute the ſlight and very faulty account here given of Mr. Horrox and his obſervations to the author's being unacquainted with his hiſtory and ſtudies. Mr. Horrox's poſthumous works, publiſhed by Dr. Wallis, would have informed him, that this incomparable youth was ſo far above the rank of an Almanack-maker, or an implicit collector from tables, that he truſted to no tables whatever, till he had corrected them from his own obſervations.—That he did not, as inſinuated, ſtumble on the tranſit of Venus through the errors of *Lanſberg*; but was led to it by his own corrections of her motions.—That from the 15th to the 23d, the laſt year of his life, he had cultivated all parts of Aſtronomy with ſuch aſſiduity and ſucceſs, that in a few years more, he muſt have carried it as far as it could go without the help of Sir Isaac Newton's principles.

HORROX's obſervations were made near *Liverpool* in *Lancaſhire*, in latitude 53° 20′, longitude weſt from *Paris* 14 minutes 15 ſeconds of time, by projecting the Sun's diſc through a teleſcope. And the concluſions he draws, which I have reduced as above to a ſolar diameter of 32′ 36″, are as follow. The differences in longitude 10′ 46″, 9′ 42″, 9′ 10″; latitude of Venus 11′ 21″, 11′ 0″, 10′ 46″, for the three inſtants above mentioned reſpectively. It was neceſſary to inſert theſe obſervations, that they may be compared with thoſe of 1761.

In the year 1691, Mr. HALLEY gave in a paper upon the tranſit of Venus (*Lowthorp*'s Abridgment, pag. 434): in which he obſerves, from the tables then in uſe, that Venus returns to a conjunction in her aſcending node in a period of 18 years wanting 2 days 10 hours 52½ minutes. But that in the ſecond conjunction ſhe will have got 24′ 41″ farther to the ſouth than in the preceding. That after a period of 235 years 2 hours 10 minutes 9 ſeconds, ſhe returns to a conjunction more to the north by 11′ 33″. And after 243 years wanting 43 minutes, in a point more to the ſouth by 13′ 8″. But if the ſecond conjunction is in the year next after a biſſextile, it will happen a day later.

The intervals of the conjunctions at the deſcending node are ſomewhat different. The ſecond happens in a period of 8 years wanting 2 days 6 hours 55 minutes; Venus being got more to the north by 19′ 58″. After 235 years 2 days 8 hours 18 minutes, ſhe is 9′ 21″ more ſoutherly: if the firſt year is a biſſextile, a day muſt be added. In fine, after 243 years 0 days 1 hour 23 minutes, the conjunction happens 10′ 37″ more to the north; or a day later, if the firſt year was biſſextile. It is ſuppoſed, as in the old ſtyle, that all the centurial years are biſſextiles.

Hence Mr. HALLEY finds the years of the vulgar æra in which a tranſit may happen at the aſcending node, in the month of November, to be theſe,—918, 1161, 1396, 1631, 1639, 1874, 2109, 2117: and the tranſits in the month of May, old ſtyle, at the deſcending node, to belong to the years 1048, 1283, 1291, 1518, 1526, 1761, 1769, 1996, 2004.

In the firſt caſe, Mr. HALLEY makes the viſible inclination of Venus's orbit to be 9° 5′, her horary motion 4′ 7″. In the latter, old ſtyle, he finds the viſible inclination to be 8° 28′, and the horary motion 4′ 0″. In either caſe, the greateſt poſſible duration of a tranſit is 7 hours 56 minutes.

Mr. **Halley** could even then conclude, that if the interval in time between the two interior contacts of Venus with the Sun could be meaſured to the exactneſs of a ſecond, in two places properly ſituated, the Sun's parallax might be determined within its 500dth part. But ſeveral years after, he explained this concluſion more fully, as follows. (*Philoſ. Tranſ.* N° 348, pag. 454, or *Jones*'s Abridgment, pag. 230.)

[*Having already inſerted the Doctor's whole Diſſertation here mentioned (in* Article VI.) *with ſome explanatory notes, there is no occaſion for repeating it again; and therefore, we ſhall proceed to the reſt of the memoir in the* Connoiſſance des Temps.]

M. **De L'Isle** has uſed the ſame method of projection to find the effect of the parallaxes in the tranſits of Mercury (*Mem. de l'Acad.* 1743, *pag.* 419.)

The calculations at the beginning of this book * differ much from Mr. **Halley**'s, being done from his own new tables, together with the new elements of nutation and aberration, the place of the node determined by M. *De la Caille*, in the year 1747, and its annual preceſſion of 19″.12, from the attraction of the other planets, as computed by myſelf † (*Mem. de l'Acad.* 1754). This difference of elements requires ſeveral new operations in finding the effect of the parallaxes; for which purpoſe I ſhall uſe a method that is independent of projections, and analogous to that which I employed for computing from the obſervations of Mercury (*Mem. de l'Acad.* 1754.)

I ſuppoſe the geocentric latitude of Venus at the time of the conjunction to be 9′ 53″.1; the Sun's diameter 31′ 37″.2; that of Venus 1′ 15″;

the

	H.	M.	S.
* In pag. 5 of the *Connoiſſance des Temps*, it is thus:			
June 6. The geocentric conjunction of Venus and the Sun at in the morning, their common longitude being ♊ 15° 37′ 33″	6	25	30
Venus's center, ſeen from the Earth's center, enters the Sun's diſc	2	55	27
The neareſt diſtance of their centers, as ſeen from the Earth's center (9′ 46″.6) — —— —	6	3	29
The beginning of the egreſs, as ſeen from the Earth's center	8	59	23
as ſeen from Paris ——	9	0	16
Venus's center on the Sun's limb, as ſeen from the Earth's center	9	11	31
as ſeen from Paris	9	12	37
Total egreſs, as ſeen from the Earth's center ——	9	23	22
as ſeen from Paris —— —— —	9	24	38

† M. *De la Lande*, of the Royal Academy of Sciences at *Paris*.

the Sun's parallax 10″; that of Venus 35″.13; the difference of the parallaxes 25″.13; half the time of the tranſit, ſeen from the Earth's center, 3 hours 8 minutes 2 ſeconds. Having computed the angle of the vertical circle with the meridian, for the given time and place, I add to it 6° 9′ for the angle which the meridian makes with the circle of latitude in the forenoon, and ſubtract the ſame angle in the afternoon; the ſum or difference being the angle which the vertical makes with the circle of latitude. And comparing this angle with that of the orbit and circle of latitude, which is 98° 30′ for the ingreſs, and 81° 30′ for the egreſs, I get the angle of the vertical with the orbit; from whence it will be eaſy to deduce Venus's parallax from the Sun, both upon the orbit itſelf and a line perpendicular to it. The former, which I call the parallax of longitude, will retard both the ingreſs and egreſs if they happen before noon, and will accelerate them if they happen in the afternoon; each ſecond of longitudinal parallax producing 15.0654 ſeconds of time, and ſo proportionably. The parallax of latitude will ſhorten or lengthen the chord which the planet deſcribes on the Sun's diſc, and conſequently the ſemiduration of the tranſit; every ſecond of this parallax will make a difference of $12\frac{11}{100}$ ſeconds in the ſemiduration; and this difference will be of the ſame denomination as the former, if the vertical paſſes between the orbit of Venus and the circle of latitude on the Sun's diſc.

In the latitude of 56° in America *, the ingreſs and egreſs happening about 65° from the meridian, the angle of the vertical with the circle of declination will be 30° 38′. ~~In the evening we~~ find the angle of the vertical with the orbit to be 135° 17′; the parallax of longitude 17″.59, its effect in time 3 minutes 22 ſeconds; the effect of the parallax of latitude, of a contrary denomination, 41′ 27″: and thence 55 ſeconds for the acceleration of the ingreſs. For the egreſs next morning, we have 105° 59′ for the angle of the vertical with the orbit; 24″.03 of longitudinal parallax, in time 4 minutes 50 ſeconds; the effect of the parallax of latitude 1 minute 44 ſeconds of time, of a contrary denomination. The egreſs therefore falls later by 3 minutes 6 ſeconds, and the total effect of the parallaxes increaſes the time of the tranſit by 4 minutes 1 ſecond.

Towards the mouth of the *Ganges*, in latitude 22°, ſuppoſing the middle of the tranſit to be at noon, the ingreſs and egreſs will fall 3 hours 20 minutes diſtance from it: and thence the angle of the vertical with the meridian will be 79° 35′, that of the vertical and orbit in the forenoon 25° 6′; the effect of the parallax of longitude 4 minutes 24 ſeconds, that of the parallax of latitude 1 minute 39 ſeconds, and conſequently the ingreſs will be retarded by 6 minutes 3 ſeconds. In the afternoon, the angle of the vertical and orbit is 4° 14′; the vertical falling not between the orbit and circle of latitude: ſo that the effect of the parallax of longitude, which is 4 minutes 50 ſeconds, is diminiſhed 17 ſeconds by the parallax of latitude. The egreſs therefore is accelerated by 4 minutes 33 ſeconds; and the total diminution of the time of the tranſit is 10 minutes 36 ſeconds *.

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* I imagine the place here meant is *Port-Nelſon* in *North America*.

It may be difficult to ſend obſervers to 6 hours, or 90 degrees diſtance to the eaſt or weſt of the meridian of *Paris*; but it may do tolerably well if they go to *Pondichery* or *Madras*, and better ſtill if an Aſtronomer takes his ſtation in the more northern parts of *America*; or at leaſt in the dominions of *Ruſſia* or *Sweden*, beyond the polar circle; that both the beginning and the end of the tranſit may be ſeen from two very diſtant places.

The countries which our calculus points out, as the moſt advantageouſly ſituated for this purpoſe, are the one in 110 degrees of longitude, not far from the tropic; the other in 290 degrees of longitude, and above 60 degrees of latitude. For, although my numbers give 58° for the latitude in which the ingreſs and egreſs may be firſt ſeen, to ſtop there would be truſting too much to a calculation. We have indeed few places properly ſituated, but the northern coaſts of *Hudſon's Bay*, the weſtern coaſts of *Baffin's Bay*. *Repulſe Bay* diſcovered in 1742, and ſome coaſts almoſt entirely unknown.—It is pity that we cannot puſh on to the polar circle, by the north of *Canada!* But by good luck, whatever places in the frigid zone, and in the Indies, our Aſtronomers chooſe to obſerve in, there will ſtill be ſome minutes of time for the effect of the parallax; to determine which, this is the fineſt opportunity that can be deſired.

In our latitude of *Paris* I found the angle of the vertical with Venus's orbit 62° 32′, the parallax of altitude 17″.44; the parallax perpendicular to the orbit 15″.47; the parallax in the orbit 8″.05: the effect of the firſt is 3 minutes 8 ſeconds, that of the ſecond 2 minutes 1 ſecond; ſo that there remains 1 minute 7 ſeconds for the retardation of the egreſs of Venus's center at *Paris*, cauſed by the parallax. The foremoſt ſemidiameter of Venus, as ſeen from the Earth's center, would take 12 minutes 8.3 ſeconds to move off the Sun's diſc; and at *Paris* the parallax adds to this time 12.6 ſeconds. The hindmoſt ſemidiameter would require 11 minutes 50.7 ſeconds, to which the parallax adds 10.8 ſeconds. And therefore, from the interior contact to the egreſs of the center, there are 12 minutes 20.9 ſeconds, and from this to the total egreſs, 12 minutes 1.5 ſeconds. Every ſecond of Venus takes 19.187 ſeconds of time to clear itſelf of the Sun's diſc; but if her heliocentric latitude ſhould be only 4″ more than we have ſuppoſed it, the geocentric latitude would be increaſed 10″; and the time of the egreſs would be increaſed by 15.6 ſeconds. And the ſame way, one ſecond of heliocentric latitude would produce a difference of 53.6 ſeconds in the duration of the tranſit.

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* It is plain that M. *De la Lande* here reckons the whole duration of the tranſit to be from the moment that Venus firſt touches the Sun's eaſtern limb to the moment of her total exit from his weſtern; otherwiſe, the whole effect of the parallaxes could not poſſibly be ſo great as he makes it; as he ſuppoſes the Sun's horizontal parallax to be only 10″. *See* § 70, in which the duration is only reckoned between the two internal contacts.

In the foregoing calculations it is ſuppoſed that the logarithm of the Sun's diſtance from the Earth is 5.006663; that of Venus's diſtance from the Sun 4.861192; the Sun's diurnal motion 57′ 21.3″; that of Venus, 37′ 31″; Venus's diurnal geocentric motion in latitude 14′ 9″; the true inclination of her orbit 3° 53′ 20″; the apparent inclination referred to the Sun's center computed as at reſt, 8° 30′ 10″, which gains the apparent inclination with reſpect to the fixed ſtars 20° 40′. The aberration of Venus in this obſervation will be 1″.4 in latitude, and 3″.7 in longitude; it will increaſe her apparent longitude, and diminiſh her latitude.

Next to the obſervations of the ingreſs and egreſs, the moſt important is that of the neareſt diſtance of the centers of Venus and the Sun; which, at *Paris* will happen about 6 in the morning: the ***Heliometer*** will be the beſt inſtrument for this purpoſe. When this diſtance is known, it will be eaſy to find the true geocentric latitude at the time of conjunction. The logarithm of this latitude, reduced to ſeconds, being added to the conſtant logarithm 0.82807, gives the logarithm of the node's diſtance; and the diſtance of the node taken from 2^s 15° 37′ 33″, gives the true place of the node. This method is more exact and ſure than thoſe which authors have uſed.

As to the other ſituations of Venus, one may meaſure the differences of her right aſcenſion and of her declination, her different altitudes and azimuths, or her neareſt diſtance from the Sun's limb. If one had only a plain perſpective glaſs, it might be employed uſefully, by fixing a circle of card-paper to it, and hanging a plummet between the paper and the perſpective. For, the Sun being thus thrown on the paper in a darkened room, will give the diſtance of Venus from the vertical, and from the Sun's center.

To conclude, another tranſit of Venus is expected on the 3d of June 1769. That and this were computed by M. *Le Gentil* (Mem. 1753). But after this tranſit of 1769, 105 years muſt elapſe before we can expect to ſee the like phenomenon.

This Author has fallen into an error of half an hour in his calculation of the time of Venus's conjunction with the Sun. He has likewiſe miſtaken the effect ariſing from the difference of Venus's parallax of longitude and her parallax of latitude. Both theſe errors together make the end of the tranſit at Paris about 32 minutes later than it ought to be by calculation.

~~ERRATA.~~

~~Pag. 13. line 2 from the bottom, *for* diſtances *read* diſtance. Pag. 14. line 19. for *dVS* read *eVS*; and for *aVC* read *AVC*. Pag. 40. line 7. *for* 18 minutes *read* 19 minutes. Pag. 45. line 16 from the bottom, *for* in *read* into.~~

FINIS.

MVSEVM BRITANNICVM

DIRECTIONS for placing the COPPER-PLATES.

Plate	fronting	Page
I.	—	7.
II.	—	17.
III.	—	29.
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